**University of Debrecen**
**Faculty of Science and Technology**
**Institute of Mathematics**

# MATHEMATICS BSC PROGRAM

**2021**

# TABLE OF CONTENTS


**DEAN`S WELCOME** ………………………………………………………………. 3
**UNIVERSITY OF DEBRECEN** …………………………………………………… 4
**FACULTY OF SCIENCE AND TECHNOLOGY** ……………………………….….. 5
**DEPARTMENTS OF INSTITUTE OF MATHEMATICS** ……………………… 6
**ACADEMIC CALENDAR** ……………………………………………………………. 8
**THE MATHEMATICS BACHELOR PROGRAM** ……………..……………….. 9
**Information about Program** ……………………………………………………… 9
**Completion of the Academic Program** …………………………………………… 11
The Credit System ……………………………………………………………... 11
Model Curriculum of Mathematics BSc Program ………………………………. 12
Work and Fire Safety Course ……………………………………………………... 15
Physical Education ……………………………………………………………….. 15
Pre-degree certification ……………………………………………………….... 15
Thesis ……………………………………………………………………………….. 15
Final Exam …………………………………………………………………………….. 16
**Diploma** ……………………………………………………………………………. 17
**Course Descriptions of Mathematics BSc Program** …………………………….. 18

# DEAN`S WELCOME

Welcome to the Faculty of Science and Technology!

This is an exciting time for you, and I encourage you to take advantage of all that the Faculty of Science and Technology UD offers you during your bachelor's or master's studies. I hope that your time here will be both academically productive and personally rewarding

Being a regional centre for research, development and innovation, our Faculty has always regarded training highly qualified professionals as a priority. Since the establishment of the Faculty in 1949, we have traditionally been teaching and working in all aspects of Science and have been preparing students for the challenges of teaching. Our internationally renowned research teams guarantee that all students gain a high quality of expertise and knowledge. Students can also take part in research and development work, guided by professors with vast international experience.

While proud of our traditions, we seek continuous improvement, keeping in tune with the challenges of the modern age. To meet the demand of the job market for professionals, we offer engineering courses with a strong scientific basis, thus expanding our training spectrum in the field of technology. Based on the fruitful collaboration with our industrial partners, recently, we successfully introduced dual-track training programmes in our constantly evolving engineering courses.

We are committed to providing our students with valuable knowledge and professional work experience, so that they can enter the job market with competitive degrees. To ensure this, we maintain a close relationship with the most important national and international companies. The basis for our network of industrial relationships are in our off-site departments at various different companies, through which market participants - future employers - are also included in the development and training of our students.

Prof. dr. Ferenc Kun
Dean

# UNIVERSITY OF DEBRECEN

**Date of foundation**: 1912 Hungarian Royal University of Sciences, 2000 University of Debrecen

**Legal predecessors**: Debrecen University of Agricultural Sciences; Debrecen Medical University; Wargha István College of Education, Hajdúböszörmény; Kossuth Lajos University of Arts and Sciences

**Number of Faculties at the University of Debrecen**: 14

Faculty of Agricultural and Food Sciences and Environmental Management

Faculty of Child and Special Needs Education

Faculty of Dentistry

Faculty of Economics and Business

Faculty of Engineering

Faculty of Health

Faculty of Humanities

Faculty of Informatics

Faculty of Law

Faculty of Medicine

Faculty of Music

Faculty of Pharmacy

Faculty of Public Health

Faculty of Science and Technology

**Number of students at the University of Debrecen**: 29,045

**Full time teachers of the University of Debrecen:** 1,541

200 full university professors and 1,205 lecturers with a PhD.

# FACULTY OF SCIENCE AND TECHNOLOGY

The Faculty of Science and Technology is currently one of the largest faculties of the University of Debrecen with about 3000 students and more than 200 staff members. The Faculty has got 6 institutes: Institute of Biology and Ecology, Institute of Biotechnology, Institute of Chemistry, Institute of Earth Sciences, Institute of Physics and Institute of Mathematics. The Faculty has a very wide scope of education dominated by science and technology (10 Bachelor programs and 12 Master programs), additionally it has a significant variety of teachers' training programs. Our teaching activities are based on a strong academic and industrial background, where highly qualified teachers with a scientific degree involve student in research and development projects as part of their curriculum. We are proud of our scientific excellence and of the application-oriented teaching programs with a strong industrial support. The number of international students of our faculty is continuously growing (currently ~650 students). The attractiveness of our education is indicated by the popularity of the Faculty in terms of incoming Erasmus students, as well.

## THE ORGANIZATIONAL STRUCTURE OF THE FACULTY

Dean: Prof. Dr. Ferenc Kun, Full Professor
E-mail: ttkdekan@science.unideb.hu

Vice Dean for Educational Affairs: Prof. Dr. Gábor Kozma, Full Professor
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Vice Dean for Scientific Affairs: Prof. Dr. Sándor Kéki, Full Professor
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Consultant on External Relationships: Prof. Dr. Attila Bérczes, Full Professor
E-mail: berczesa@science.unideb.hu

Dean's Office
Head of Dean's Office: Ms. Katalin Tóth
E-mail: toth.katalin@science.unideb.hu

English Program Officer: Mr. Imre Varga – Applied Mathematics (MSc), Chemical Engineering (BSc/MSc), Chemistry (BSc/MSc), Earth Sciences (BSc), Electrical Engineering (BSc), Geography (BSc/MSc), Mathematics (BSc), Physics (BSc), Physicist (MSc), International Foundation Year, Intensive Foundation Semester
Address: 4032 Egyetem tér 1., Chemistry Building, A/101
E-mail: vargaimre@unideb.hu

English Program Officer: Mrs. Szilvia Gyulainé Szemerédi – Biochemical Engineering (BSc), Biology (BSc/MSc), Envirionmental Science (MSc), Hidrobiology Water Quality Management (MSc)
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# DEPARTMENTS OF INSTITUTE OF MATHEMATICS

**Department of Algebra and Number Theory** (home page: http://math.unideb.hu/algebra/en)
**4032 Debrecen, Egyetem tér 1, Geomathematics Building**

| Name | Position | E-mail | room |
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| Mr. Prof. Dr. Attila Bérczes | University Professor, Head of Department | berczesa@science.unideb.hu | M415 |
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| Mr. Dr. Zoltán Kovács | Associate Professor | kovacsz@science.unideb.hu | M303 |
| Mr. Dr. László Kozma | Associate Professor | kozma@unideb.hu | M306 |
| Mr. Dr. Csaba Vincze | Associate Professor, Deputy Director of Institute | csvincze@science.unideb.hu | M304 |
| Mr. Dr. Tran Quoc Binh | Senior Research Fellow | binh@science.unideb.hu | M305 |
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| Mr. Dr. Ábris Nagy | Assistant Lecturer | abris.nagy@science.unideb.hu | M304 |
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## ACADEMIC CALENDAR

General structure of the academic semester (2 semesters/year):

| | | | |
|---|---|---|---|
| Study period | 1st week | Registration* | 1 week |
| | 2nd – 15th week | Teaching period | 14 weeks |
| Exam period | directly after the study period | Exams | 7 weeks |

*Usually, registration is scheduled for the first week of September in the fall semester, and for the first week of February in the spring semester.

For further information please check the following link:

https://www.edu.unideb.hu/tartalom/downloads/University_Calendars_2021_22/University_calendar_2021-2022-Faculty_of_Science_and_Technology.pdf?_ga=2.196279020.1315409739.1629100510-488342717.1574682820

# THE MATHEMATICS BACHELOR PROGRAM

## Information about the Program

| | |
|---|---|
| Name of BSc Program: | Mathematics BSc Program |
| Specialization available: | |
| Field, branch: | Science |
| Qualification: | Mathematician |
| Mode of attendance: | Full-time |
| Faculty, Institute: | Faculty of Science and Technology<br>Institute of Mathematics |
| Program coordinator: | Prof. Dr. György Gát, University Professor |
| Duration: | 6 semesters |
| ECTS Credits: | 180 |

**Objectives of the BSc program:**

The aim of the Mathematics BSc program is to train professional mathematicians who have deep knowledge on theoretical and applied mathematics that makes them capable of using their basic mathematical knowledge on the fields of engineering, economics, statistics and informatics. They are prepared to continue to study in an MSc program.

**Professional competences to be acquired**

**A Mathematician:**

**a) Knowledge:**

- He/she knows the basic methods of mathematics in the fields of analysis, algebra, geometry, discrete mathematics, operations research and probability theory (statistics).
- He/she knows the basic correlations in pure mathematics, related to the fields of analysis, algebra, geometry, discrete mathematics, operations research and probability theory (statistics).
- He/she knows the basic correlations between different subdisciplines of mathematics.
- He/she is aware of the requirements of defining abstract concepts, he/she recognises general patterns and concepts inherited in the problems applied.
- He/she knows the requirements and basic methods of mathematical proofs.
- He/she is aware of the specific features of mathematical thinking.

**b) Abilities:**

- He/she is capable of formulating and communicating true and logical mathematical statements, as well as, how to exactly indicate their conditions and main consequences.
- He/she is capable of drawing conclusions of the qualitative type from quantitative data.
- He/she is capable of applying his/her factual knowledge acquired in the fields of analysis, algebra, geometry, discrete mathematics, operations research and probability theory (statistics).

- He/she is capable of finding and exploring new correlations in the fields of analysis, algebra, geometry, discrete mathematics, operations research and probability theory (statistics).
- He/she is capable of going beyond the concrete forms of problems, and formulating them both in abstract and general forms for the sake of analysis and finding a solution.
- He/she is capable of designing experiments for the sake of data collection, as well as, of analysing the results achieved by the means of mathematics and informatics.
- He/she is capable of making a comparative analysis of different mathematical models.
- He/she is capable of effectively communicating the results of mathematical analyses in foreign languages, and by the means of informatics.
- He/she is capable of identifying routine problems of his/her own professional field, using the scientific literature available (library and electronic sources) and adapting their methods to find theoretical and practical solutions

**c) Attitude:**
- He/she desires to enhance the scope of his/her mathematical knowledge by learning new concepts, as well as, for acquiring and developing new competencies.
- He/she aspires to apply his/her mathematical knowledge as widely as possible.
- Applying his/her mathematical knowledge, he/she aspires to get acquainted with the perceptible phenomena in the most thorough way possible, and to describe and explain the principles shaping them.
- Using his/her mathematical knowledge, he/she aspires to apply scientific reasoning.
- He/she is open to recognizing the specific problems in professional fields other than his/her own field and makes an effort to cooperate with experts of these fields, to the end of proposing a mathematical adaptation of field-specific problems.
- He/she is open to continuing professional training and development in the field of mathematics.

**d) Autonomy and responsibility:**
- Using his/her basic knowledge acquired in mathematical subdisciplines, he/she is capable of formulating and analysing mathematical questions on his/her own.
- He/she responsibly assesses mathematical results, their applicability and the limits of their applicability.
- He/she is aware of the value of mathematical-scientific statements, their applicability and the limits of their applicability.
- He/she is capable of making decisions on his/her own, based on the results of mathematical analyses.
- He/she is aware that he/she must carry out his/her own professional work in line with the highest ethical standards and ensuring a high level of quality.
- He/she carries out his/her theoretical and practical research activities related to different fields of mathematics, with the necessary guidance, on his/her own.

# Completion of the BSc Program

*The Credit System*

Majors in the Hungarian Education System have generally been instituted and ruled by the Act of Parliament under the Higher Education Act. The higher education system meets the qualifications of the Bologna Process that defines the qualifications in terms of learning outcomes: statements of what students know and can do on completing their degrees. In describing the cycles, the framework uses the European Credit Transfer and Accumulation System (ECTS).
ECTS was developed as an instrument of improving academic recognition throughout the European Universities by means of effective and general mechanisms. ECTS serves as a model of academic recognition, as it provides greater transparency of study programs and student achievement. ECTS in no way regulates the content, structure and/or equivalence of study programs.
Regarding each major the Higher Education Act prescribes which professional fields define a certain training program. It contains the proportion of the subject groups: natural sciences, economics and humanities, subject-related subjects and differentiated field-specific subjects.

During the program students have to complete a total amount of 120 credit points. It means approximately 30 credits per semester. The curriculum contains the list of subjects (with credit points) and the recommended order of completing subjects which takes into account the prerequisite(s) of each subject. You can find the recommended list of subjects/semesters in chapter “Model Curriculum of Mathematics BSc Program”.

*Model Curriculum of Mathematics BSc Program*

| | semesters | | | | | | ECTS credit points | evaluation |
|---|---|---|---|---|---|---|---|---|
| | 1. | 2. | 3. | 4. | 5. | 6. | | |
| | contact hours, types of teaching (l – lecture, p – practice), credit points | | | | | | | |
| **Linear algebra subject group** | | | | | | | | |
| Linear algebra 1. *Dr. Gaál István* | 28 l/3 cr. 28 p /2 cr. | | | | | | *5* | exam mid-semester grade |
| Linear algebra 2. *Dr. Gaál István* | | 28 l/3 cr. 28 p/2 cr. | | | | | *5* | exam mid-semester grade |
| **Classical algebra subject group** | | | | | | | | |
| Introduction to Algebra and Number Theory *Dr. Pintér Ákos* | 28 l/3 cr. 42 p/2 cr. | | | | | | *6* | exam mid-semester grade |
| Algebra 1. *Dr. Szikszai Márton* | | 28 l/3 cr. 28 p/2 cr. | | | | | *5* | exam mid-semester grade |
| Algebra 2. *Dr. Szikszai Mártonr* | | | 28 l/3 cr. 28 p/2 cr. | | | | *5* | exam mid-semester grade |
| **Classical finite mathematics subject group** | | | | | | | | |
| Number theory *Dr. Hajdu Lajos* | | | 28 l/3 cr. 28 p/2 cr. | | | | *5* | exam mid-semester grade |
| Combinatorics and graph theory *Dr. Nyul Gábor* | 42 l/4 cr . 28 p/2 cr. | | | | | | *6* | exam mid-semester grade |
| **Classical analysis subject group** | | | | | | | | |
| Sets and functions *Dr. Lovas Rezső* | 28 l/3 cr. 28 p/2 cr. | | | | | | *5* | exam mid-semester grade |
| Introduction to analysis *Dr. Bessenyei Mihály* | | 42 l/4 cr. 28 p/2 cr. | | | | | *6* | exam mid-semester grade |
| Differential and integral calculus *Dr. Bessenyei Mihály* | | | 42 l/4 cr. 42 p/3 cr. | | | | *7* | exam mid-semester grade |
| Differential and integral calculus in several variables *Dr. Páles Zsolt* | | | | 42 l/4 cr . 42 p/3 cr. | | | *7* | exam mid-semester grade |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Ordinary differential equations<br>*Dr. Gát György* | | | | | 28 l/3 cr.<br>28 p/2 cr. | | *5* | exam<br>mid-semester grade |
| **Classical geometry subject group** | | | | | | | | |
| Geometry 1.<br>*Dr. Vincze Csaba* | 28 l/3 cr.<br>28 p/2 cr. | | | | | | *5* | exam<br>mid-semester grade |
| Geometry 2.<br>*Dr. Vincze Csaba* | | 28 l/3 cr.<br>28 p/2 cr. | | | | | *5* | exam<br>mid-semester grade |
| Differential geometry<br>*Dr. Muzsnay Zoltán* | | | | | 28 l/3 cr.<br>28 p/2 cr. | | *5* | exam<br>mid-semester grade |
| Vector analysis<br>*Dr. Vincze Csaba* | | | | | | 28 l/3 cr.<br>28 p/2 cr. | *5* | exam<br>mid-semester grade |
| **Probability theory subject group** | | | | | | | | |
| Measure and integral theory<br>*Dr. Nagy Gergő* | | | | 28 l/3 cr. | | | *3* | exam |
| Probability theory<br>*Dr. Fazekas István* | | | | | 42 l/4 cr.<br>28 p/2 cr. | | *6* | exam<br>mid-semester grade |
| Statistics<br>*Dr. Barczy Mátyás* | | | | | | 42 l/4 cr.<br>28 p/2 cr. | *5* | exam<br>mid-semester grade |
| **Informatics subject group** | | | | | | | | |
| Introduction to informatics<br>*Dr. Tengely Szabolcs* | 42 p/2 cr. | | | | | | *2* | mid-semester grade |
| Progamming languages<br>*Dr. Bazsó András* | 28 p/2 cr. | | | | | | *2* | mid-semester grade |
| **Finite mathematical algorithms subject group** | | | | | | | | |
| Algorithms<br>*Dr. Györkös-Varga Nóra* | | 28 l/3 cr.<br>28 p/2 cr. | | | | | *5* | exam<br>mid-semester grade |
| Applied number theory<br>*Dr. Hajdu Lajos* | | | | 42 l/3 cr. | | | *3* | exam |
| Algorithms in algebra and number theory<br>*Dr. Tengely Szabolcs* | | | | 42 p/3 cr. | | | *3* | mid-semester grade |
| Introduction to cryptography<br>*Dr. Bérczes Attila* | | | | | 28 l/3 cr.<br>28 p /2 cr. | | *5* | exam<br>mid-semester grade |
| **Applied analysis subject group** | | | | | | | | |
| Numerical analysis<br>*Dr. Fazekas Borbála* | | | | 42 l/4 cr.<br>28 p/2 cr. | | | *6* | exam<br>mid-semester grade |

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|---|---|---|---|---|---|---|---|---|
| Economic mathematics<br>*Dr. Mészáros Fruzsina* | | | | | | 28 l/3 cr.<br>28 p/2 cr. | *5* | exam<br>mid-semester grade |
| **Computer mathematics subject group** | | | | | | | | |
| Analysis with computer<br>*Dr. Fazekas Borbála* | | | | | | 42 p/3 cr. | *3* | mid-semester grade |
| Computer statistics<br>*Dr. Sikolya-Kertész Kinga* | | | | | | 28 p/2 cr. | *2* | mid-semester grade |
| Computer geometry<br>*Dr. Nagy Ábris* | | | 42 p/3 cr. | | | | *3* | mid-semester grade |
| **Optimizing subject group** | | | | | | | | |
| Linear programming<br>*Dr. Mészáros Fruzsina* | | | 28 l/3 cr.<br>28 p/2 cr. | | | | *5* | exam<br>mid-semester grade |
| Nonlinear optimization<br>*Dr. Páles Zsolt* | | | | | 28 l/3 cr.<br>28 p/2 cr. | | *5* | exam<br>mid-semester grade |
| **Basics of earth sciences and mathematics subject group** | | | | | | | | |
| Basics of mathematics<br>*Dr.Györkös- Varga Nóra* | 14 p/0 cr. | | | | | | *0* | signature |
| Classical mechanics<br>*Dr. Erdélyi Zoltán* | | | | 28 l/3 cr.<br>14 p/1 cr. | | | *4* | exam |
| Theoretical mechanics<br>*Dr. Nagy Sándor* | | | | | | 28 l/3 cr.<br>14 p/1 cr. | *4* | exam |
| European Union studies<br>*Dr. Teperics Károly* | 14 p/1 cr. | | | | | | *1* | exam |
| Basic environmental science<br>*Dr. Nagy Sándor Alex* | 14 p/1 cr. | | | | | | *1* | exam |
| **Thesis I.** | | | | | 5 cr. | | *5* | mid-semester grade |
| **Thesis II.** | | | | | | 5 cr. | **5** | mid-semester grade |
| *optional courses* | | | | | | | | |
| optional courses | | | | | | | *9* | |

## *Work and Fire Safety Course*

According to the Rules and Regulations of University of Debrecen a student has to complete the online course for work and fire safety. Registration for the course and completion are necessary for graduation. For MSc students the course is only necessary only if BSc diploma has been awarded outside of the University of Debrecen.
Registration in the Neptun system by the subject: MUNKAVEDELEM
Students have to read an online material until the end to get the signature on Neptun for the completion of the course. The link of the online course is available on webpage of the Faculty.

## *Physical Education*

According to the Rules and Regulations of University of Debrecen a student has to complete Physical Education courses at least in two semesters during his/her Bachelor’s training. Our University offers a wide range of facilities to complete them.

## *Pre-degree Certification*

A pre-degree certificate is issued by the Faculty after completion of the bachelor’s (BSc) program. The pre-degree certificate can be issued if the student has successfully completed the study and exam requirements as set out in the curriculum, the requirements relating to Physical Education as set out in Section 10 in Rules and Regulations, internship (mandatory) – with the exception of preparing thesis – and gained the necessary credit points (180). The pre-degree certificate verifies (without any mention of assessment or grades) that the student has fulfilled all the necessary study and exam requirements defined in the curriculum and the requirements for Physical Education. Students who obtained the pre-degree certificate can submit the thesis and take the final exam.

## *Thesis*

Students have to choose a topic for their thesis two semesters before the expected date of finishing their studies, i.e., usually at the end of the 4th semester. They have to write it in two semesters, and they have to register for the courses ‘Thesis 1’ and ‘Thesis 2’ in two different semesters. They write the thesis with the help of a supervisor who should be a lecturer of the Institute of Mathematics. (In exceptional cases, the supervisor can be a member of another institute.)
Students are not required to present new scientific results, but they have to do some scientific work on their own. The thesis should be about 20–40 pages long and using the LaTeX document preparation system is recommended. The cover page has to contain the name of the institute, the title of the thesis, the name and the degree program of the student, the name and the university rank of the supervisor. Besides the detailed discussion of the topic, the thesis should

contain an introduction, a table of contents and a bibliography. The thesis has to be defended in the final exam.

## *Final Exam*

The final exam is an oral exam before a committee designated by the Director of the Institute of Mathematics and approved by the leaders of the Faculty of Science and Technology. The final exam consists of two parts: an account by the student on a certain exam question, and the defense of the thesis. The questions of the final exam comprise the compulsory courses of the Mathematics BSc Program. Students draw a random question from the list, and after a certain preparation period, give an account on it. After this, the committee may ask questions also from other topics. Students get three separate marks for their answers on the exam question, for the thesis and for the defense of the thesis.

Final Exam Board
Board chair and its members are selected from the acknowledged internal and external experts of the professional field. Traditionally, it is the chair and in case of his/her absence or indisposition the vice-chair who will be called upon, as well. The board consists of – besides the chair – at least two members (one of them is an external expert), and questioners as required. The mandate of a Final Examination Board lasts for one year.

Repeating a failed Final Exam
If any part of the final exam is failed it can be repeated according to the rules and regulations. A final exam can be retaken in the forthcoming final exam period. If the Board qualified the Thesis unsatisfactory a student cannot take the final exam and he has to make a new thesis. A repeated final exam can be taken twice on each subject.

# Diploma

The diploma is an official document decorated with the coat of arms of Hungary which verifies the successful completion of studies in the Mathematics Bachelor Program. It contains the following data: name of HEI (higher education institution); institutional identification number; serial number of diploma; name of diploma holder; date and place of his/her birth; level of qualification; training program; specialization; mode of attendance; place, day, month and year issued. Furthermore, it has to contain the rector's (or vice-rector's) original signature and the seal of HEI. The University keeps a record of the diplomas issued.

In Mathematics Bachelor Master Program the diploma grade is calculated as the average grade of the results of the followings:

- Weighted average of the overall studies at the program (A)
- Average of grades of the thesis and its defense given by the Final Exam Board (B)
- Average of the grades received at the Final Exam for the two subjects (C)

Diploma grade = (A + B + C)/3

Classification of the award on the bases of the calculated average:

| | |
|---|---|
| Excellent | 4.81 – 5.00 |
| Very good | 4.51 – 4.80 |
| Good | 3.51 – 4.50 |
| Satisfactory | 2.51 – 3.50 |
| Pass | 2.00 – 2.50 |

# Course Descriptions of Mathematics BSc Program

| | |
|---|---|
| **Title of course**: Linear algebra 1.<br>**Code**: TTMBE0102 | **ECTS Credit points:** 3 |

**Type of teaching, contact hours**
- lecture: 2 hours/week
- practice: -
- laboratory: -

**Evaluation:** exam

**Workload (estimated), divided into contact hours:**
- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 62 hours

Total: 90 hours

**Year, semester**: 1st year, 1st semester

**Its prerequisite(s)**: -

**Further courses built on it**: TTMBE0103, TTMBE0607, TTMBE0209, TTMBG0701

**Topics of course**

Basic notions in algebra. Determinants. Operations with matrices. Vector spaces, basis, dimension. Linear mappings. Transformation of basis and coordinates. The dimensions of the row space and the column space of matrices are equal. Sum and direct sum of subspaces. Factor spaces. Systems of linear equations. Matrix of a linear transformation. Operations with linear transformations. Similar matrices. Eigenvalues, eigenvectors. Characteristic polynomial. The existence of a basis consisting of eigenvectors.

**Literature**

*Compulsory:*

-

*Recommended:*

Paul R. Halmos: Finite dimensional vector spaces, Benediction Classics, Oxford, 2015.

Serge Lang, Linear Algebra, Springer Science & Business Media, 2013.

Howard Anton and Chris Rorres, Elementary Linear Algebra, John Wiley & Sons, 2010.

**Schedule:**

*1st week*

Basic concepts of algebra. Permutations and their properties.

*2nd week*

Determinants. Expanding determinants. Laplace expansion theorem.

*3rd week*

Operations on matrices. Matrix algebra. Multiplication theorem of determinants. Inverse of matrices.

*4th week*

Vector space, subspace, generating system, linear dependence and independence. Basis, dimension.

*5th week*

Linear mappings of vector spaces. Fundamental theorems on linear mappings. Transformation of bases and coordinates.

*6th week*

Rank of a set of vectors, rank of a matrix. Theorem on ranks. Calculating the rank of a matrix by elimination.

*7th week*

Sum and direct sum of subspaces. Equivalent properties. Coset of subspaces. Factor spaces of vector spaces. Dimension of the factor space.

*8th week*

Systems of linear equations. Criteria for solubility, for the uniqueness of solutions. Homogeneous systems of linear equations. Solutions space, the dimension of the solution space.

*9th week*

Inhomogeneous systems of linear equations. The structure of solutions. Cramer's rule Gaussian elimination.

*10th week*

Linear mappings of vector spaces. Kernel, image. Theorem on homomorphisms. The condition of injectivity.

*11th week*

Linear transformations. Injective and surjective linear transformations. The matrix of a linear transformation. Calculation the image vector. The matrix of the linear transformation in a new basis.

*12th week*

Operations on linear transformations. Algebra of linear transformations. Similar matrices. Automorphisms.

*13th week*

Invariant subspaces. Eigenvector, eigenvalues of a linear transformation. Eigenspace. Eigen-vectors of distinct eigenvalues. Eigenspaces of distinct eigenvalues.

*14th week*

Characteristic polynomial. Algebraic and geometric multiplicity of eigenvalues. Spectrum of a linear transformation. Existence of a basis consisting of eigenvectors.

**Requirements:**

*- for a signature*

If the student fail the course TTMBG0102, then the signature is automatically denied.

*- for a grade*

The course ends in oral examination. The grade is given according to the following table:

| **Total Score (%)** | **Grade** |
|---|---|
| 0 – 50 | fail (1) |
| 51 – 60 | pass (2) |
| 61 – 70 | satisfactory (3) |
| 71 – 80 | good (4) |
| 81 – 100 | excellent (5) |

*-an offered grade:*

It is not possible to obtain an offered grade in this course.

| |
|---|
| **Person responsible for course:** Prof. Dr. István Gaál, university professor, DSc |
| **Lecturer:** Prof. Dr. István Gaál, university professor, DSc |

| **Title of course**: Linear algebra 1.<br>**Code**: TTMBG0102 | **ECTS Credit points:** 2 |
|---|---|

**Type of teaching, contact hours**
- lecture: -
- practice: 2 hours/week
- laboratory: -

**Evaluation:** mid-semester grade

**Workload (estimated), divided into contact hours:**
- lecture: -
- practice: 28 hours
- laboratory: -
- home assignment: -
- preparation for the exam: 32 hours

Total: 60 hours

**Year, semester**: 1st year, 1st semester

**Its prerequisite(s)**: -

**Further courses built on it**: -

**Topics of course**

Basic notions in algebra. Determinants. Operations with matrices. Vector spaces, basis, dimension. Linear mappings. Transformation of basis and coordinates. The dimensions of the row space and the column space of matrices are equal. Sum and direct sum of subspaces. Factor spaces. Systems of linear equations. Matrix of a linear transformation. Operations with linear transformations. Similar matrices. Eigenvalues, eigenvectors. Characteristic polynomial. The existence of a basis consisting of eigenvectors.

**Literature**

*Compulsory:*

-

*Recommended:*

Paul R. Halmos: Finite dimensional vector spaces, Benediction Classics, Oxford, 2015.
Serge Lang, Linear Algebra, Springer Science & Business Media, 2013.
Howard Anton and Chris Rorres, Elementary Linear Algebra, John Wiley & Sons, 2010.

**Schedule:**

*1st week*

Abstract groups, permutation.

*2nd week*

Determinants. Expanding determinants.

*3rd week*

Operations on matrices.

*4th week*

Inverse of matrices. Vectors spaces. Basis, dimension.

*5th week*

Transformation of bases and coordinates.

*6th week*

Rank of a matrix. Calculating the rank of a matrix by elimination.

*7th week*

First test.

*8th week*

Homogeneous systems of linear equations. Solutions space.

*9th week*

Inhomogeneous systems of linear equations. Cramer's rule Gaussian elimination.

*10th week*

Linear mappings of vector spaces. Calculating the kernel and image.

*11th week*

The matrix of a linear transformation. The matrix of the linear transformation in a new basis.

*12th week*

Operations on linear transformations. Similar matrices.

*13th week*

Able to calculate eigenvalues, eigenvectors, basis consisting of eigenvectors.

*14th week*

Second test.

**Requirements:**

*- for a signature*

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

*- for a grade*

The course is evaluated on the basis of two written tests during the semester. The grade is given according to the following table:

| **Total Score (%)** | **Grade** |
|---|---|
| 0 – 50 | fail (1) |
| 51 – 60 | pass (2) |
| 61 – 70 | satisfactory (3) |
| 71 – 80 | good (4) |
| 81 – 100 | excellent (5) |

If a student fail to pass at first attempt, then a retake of the tests is possible.

*-an offered grade:*

It is not possible to obtain an offered grade in this course.

**Person responsible for course:** Prof. Dr. István Gaál, university professor, DSc

**Lecturer:** Prof. Dr. István Gaál, university professor, DSc

| **Title of course**: Linear algebra 2.<br>**Code**: TTMBE0103 | **ECTS Credit points:** 3 |
|---|---|

**Type of teaching, contact hours**
- lecture: 2 hours/week
- practice: -
- laboratory: -

**Evaluation:** exam

**Workload (estimated), divided into contact hours:**
- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 62 hours

Total: 90 hours

**Year, semester**: 1st year, 2nd semester

**Its prerequisite(s)**: TTMBE0102

**Further courses built on it**: -

**Topics of course**

Linear forms, bilinear forms, quadratic forms. Inner product, Euclidean space. Inequalities in Euclidean spaces. Orthonormal bases. Gram-Schmidt orthogonalization method. Orthogonal complement of a subspace. Complex vectorspaces with inner product: unitary spaces. Linear forms, bilinear forms and inner products. Adjoint of a linear transformation. Properties of the adjoint transformation. Self-adjoint transformations. Isometric/orthogonal transformations. Normal transformations.

**Literature**

*Compulsory:*
-
*Recommended:*
Paul R. Halmos: Finite dimensional vector spaces, Benediction Classics, Oxford, 2015.
Serge Lang, Linear Algebra, Springer Science & Business Media, 2013.
Howard Anton and Chris Rorres, Elementary Linear Algebra, John Wiley & Sons, 2010.

**Schedule:**

*1st week*

Nilpotent transformations. Canonical form of a nilpotent matrix.

*2nd week*

Jordan normal form, Jordan blocks, canonical basis.

*3rd week*

Linear forms, bilinear forms, quadratic forms.

*4th week*

Canonical form of bilinear and quadratic forms. Lagrange theorem. Sylvester theorem. Jacobi theorem. Positive definite quadratic forms and their characterization.

*5th week*

Inner product, Euclidean space, Cauchy-Bunyakovszkij-Schwarz inequality, Minkowski inequality.

*6th week*

Gram-Schmidt orthogonalization method, orthonormed bases, orthogonal complement of a subspace, Bessel inequality, Parseval equation.

*7th week*

Bilinear and quadratic forms in complex vector spaces. Inner product. Unitary spaces.

*8th week*

Linear, bilinear forms and inner products. Adjoint transformations. Properties of the adjoint transformation.

*9th week*

Self-adjoint transformations, eigenvalues, eigenvectors, canonical form.

*10th week*

Orthogonal transformations. Equivalent properties. Properties of orthogonal matrices.

*11th week*

Orthogonal transformations of Euclidean spaces. Quasi diagonal matrices. Representation of linear transformations by self-adjoint transformations.

*12th week*

Normal transformations in unitary spaces. Polar representation theorem.

*13th week*

Curves of second order, Asymptote directions. Diameters conjugated to a direction. Principal axis. Transformation to principal axis.

*14th week*

Application of symbolic algebra packages in linear algebra calculations.

**Requirements:**

*- for a signature*

If the student fail the course TTMBG0103, then the signature is automatically denied.

*- for a grade*

The course ends in oral examination. The grade is given according to the following table:

| Total Score (%) | Grade |
|---|---|
| 0 – 50 | fail (1) |
| 51 – 60 | pass (2) |
| 61 – 70 | satisfactory (3) |
| 71 – 80 | good (4) |
| 81 – 100 | excellent (5) |

*-an offered grade:*

It is not possible to obtain an offered grade in this course.

**Person responsible for course:** Prof. Dr. István Gaál, university professor, DSc

**Lecturer:** Prof. Dr. István Gaál, university professor, DSc

| **Title of course**: Linear algebra 2.<br>**Code**: TTMBG0103 | **ECTS Credit points:** 2 |
|---|---|

**Type of teaching, contact hours**
- lecture: -
- practice: 2 hours/week
- laboratory: .

**Evaluation:** mid-semester grade

**Workload (estimated), divided into contact hours:**
- lecture: -
- practice: 28 hours
- laboratory: -
- home assignment: -
- preparation for the exam: 32 hours

Total: 60 hours

**Year, semester**: 1st year, 2nd semester

**Its prerequisite(s)**: TTMBE0102

**Further courses built on it**: -

**Topics of course**

Linear forms, bilinear forms, quadratic forms. Inner product, Euclidean space. Inequalities in Euclidean spaces. Orthonormal bases. Gram-Schmidt orthogonalization method. Orthogonal complement of a subspace. Complex vectorspaces with inner product: unitary spaces. Linear forms, bilinear forms and inner products. Adjoint of a linear transformation. Properties of the adjoint transformation. Self-adjoint transformations. Isometric/orthogonal transformations. Normal transformations.

**Literature**

*Compulsory:*

-

*Recommended:*

Paul R. Halmos: Finite dimensional vector spaces, Benediction Classics, Oxford, 2015.
Serge Lang, Linear Algebra, Springer Science & Business Media, 2013.
Howard Anton and Chris Rorres, Elementary Linear Algebra, John Wiley & Sons, 2010.

**Schedule:**

*1st week*

Nilpotent transformations.

*2nd week*

Jordan normal form.

*3rd week*

Linear forms, bilinear forms, quadratic forms.

*4th week*

Canonical form of bilinear and quadratic forms. Positive definite quadratic forms and their characterization.

*5th week*

Inner product, Euclidean space.

*6th week*

Gram-Schmidt orthogonalization method, orthonormed bases.

*7th week*

First test.

*8th week*

Adjoint transformations. Properties of the adjoint transformation.

*9th week*

Self-adjoint transformations, eigenvalues, eigenvectors, canonical form.

*10th week*

Orthogonal transformations.

*11th week*

Orthogonal transformations of Euclidean spaces. Quasi diagonal matrices.

*12th week*

Normal transformations in unitary spaces. Polar representation theorem.

*13th week*

Curves of second order. Transformation to principal axis.

*14th week*

Second test.

**Requirements:**

*- for a signature*

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

*- for a grade*

The course is evaluated on the basis of two written tests during the semester. The grade is given according to the following table:

| Total Score (%) | Grade |
|---|---|
| 0 – 50 | fail (1) |
| 51 – 60 | pass (2) |
| 61 – 70 | satisfactory (3) |
| 71 – 80 | good (4) |
| 81 – 100 | excellent (5) |

If a student fail to pass at first attempt, then a retake of the tests is possible.

*-an offered grade:*

It is not possible to obtain an offered grade in this course.

**Person responsible for course:** Prof. Dr. István Gaál, university professor, DSc

**Lecturer:** Prof. Dr. István Gaál, university professor, DSc

| **Title of course**: Introduction to algebra and number theory<br>**Code**: TTMBE0101 | **ECTS Credit points:** 3 |
|---|---|

**Type of teaching, contact hours**
- lecture: 2 hours/week
- practice: -
- laboratory: -

**Evaluation:** exam

**Workload (estimated), divided into contact hours:**
- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 62 hours

Total: 90 hours

**Year, semester**: 1st year, 1st semester

**Its prerequisite(s)**: -

**Further courses built on it**: TTMBE0104, TTMBG0701

**Topics of course**

Relations, algebraic structures, operations and their properties. Divisibility and division with remainder in Z. Greatest common divisor, Euclidean algorithm. Congruence relation and congruence classes in Z, rings of congruence classes. The theorem of Euler-Fermat. Linear congruences. Linear congruence systems, Chinese remainder theorem. Two-variable and multivariate linear Diophantine equations. Peano axioms, N, Z, Q. Complex numbers, operations, conjugate, absolute value. Trigonometric form of complex numbers, theorem of Moivre, nth roots of complex numbers, roots of unity. Polynomial ring over field. Euclidean division, greatest common divisor. Polynomial rings over Z, Q, R, and C, absolute value. Fundamental theorem of algebra. Partial fraction expression. Algebraic equations, discriminant, resultant, multiple roots, cubic and quartic equations. Multivariate polynomials, symmetric and elementary symmetric functions, fundamental theorem of symmetric polynomials.

**Literature**

*Compulsory:*

-

*Recommended:*

I. Nivan, H. S. Zuckerman, H. L. Montgomery: An introduction to the theory of numbers. John Wiley and Sons, 1991.

L. N., Childs:: A concrete introduction to higher algebra. New York, Springer, 2000.

**Schedule:**

*1st week*

Relations, algebraic structures, operations and their properties.

*2nd week*

Peano axioms, natural numbers.

*3rd week*

Integer and rational numbers.

*4th week*

Complex numbers, operations, conjugate, absolute value.

*5th week*

Trigonometric form of complex numbers, theorem of Moivre, nth roots of complex numbers, roots of unity.

*6th week*

Divisibility and division with remainder in Z. Greatest common divisor, Euclidean algorithm.

*7th week*

Congruence relation and congruence classes in Z, rings of congruence classes. Euler's phi-function, the theorem of Euler-Fermat.

*8th week*

Linear congruences. Condition of solvability, number of solutions. Linear congruence systems, Chinese remainder theorem.

*9th week*

Two-variable linear Diophantine equations, condition of solvability and their connection with linear congruences, multivariate linear Diophantine equations.

*10th week*

Polynomial ring over field. Euclidean division, greatest common divisor.

*11th week*

Ring of Z[x], Q[x], R[x], C[x], irreducible factorization.

*12th week*

Fundamental theorem of algebra. Partial fraction expression.

*13th week*

Algebraic equations, discriminant, resultant, multiple roots, cubic and quartic equations.

*14th week*

Multivariate polynomials, symmetric and elementary symmetric functions, fundamental theorem of symmetric polynomial.

**Requirements:**

*- for a signature*

If the student fail the course TTMBG0101, then the signature is automatically denied.

*- for a grade*

The course ends in oral examination. The grade is given according to the following table:

| **Total Score (%)** | **Grade** |
|---|---|
| 0 – 50 | fail (1) |
| 51 – 60 | pass (2) |
| 61 – 70 | satisfactory (3) |
| 71 – 85 | good (4) |
| 86 – 100 | excellent (5) |

*-an offered grade:*

It is not possible to obtain an offered grade in this course.

**Person responsible for course:** Prof. Dr. Ákos Pintér, university professor, DSc

**Lecturer:** Prof. Dr. Ákos Pintér, university professor, DSc

| **Title of course**: Introduction to algebra and number theory<br>**Code**: TTMBG0101 | **ECTS Credit points:** 2 |
|---|---|

**Type of teaching, contact hours**
- lecture: -
- practice: 2 hours/week
- laboratory: -

**Evaluation:** mid-semester grade

**Workload (estimated), divided into contact hours:**
- lecture: -
- practice: 28 hours
- laboratory: -
- home assignment: -
- preparation for the exam: 32 hours

Total: 60 hours

**Year, semester**: 1st year, 1st semester

**Its prerequisite(s)**: -

**Further courses built on it**: -

**Topics of course**

Relations, algebraic structures, operations and their properties. Divisibility and division with remainder in Z. Greatest common divisor, Euclidean algorithm. Congruence relation and congruence classes in Z, rings of congruence classes. The theorem of Euler-Fermat. Linear congruences. Linear congruence systems, Chinese remainder theorem. Two-variable and multivariate linear Diophantine equations. Peano axioms, N, Z, Q. Complex numbers, operations, conjugate, absolute value. Trigonometric form of complex numbers, theorem of Moivre, nth roots of complex numbers, roots of unity. Polynomial ring over field. Euclidean division, greatest common divisor. Polynomial rings over Z, Q, R, and C, absolute value. Fundamental theorem of algebra. Partial fraction expression. Algebraic equations, discriminant, resultant, multiple roots, cubic and quartic equations. Multivariate polynomials, symmetric and elementary symmetric functions, fundamental theorem of symmetric polynomials.

**Literature**

*Compulsory:*

-

*Recommended:*

I. Nivan, H. S. Zuckerman, H. L. Montgomery: An introduction to the theory of numbers. John Wiley and Sons, 1991.

L. N., Childs:: A concrete introduction to higher algebra. New York, Springer, 2000.

**Schedule:**

*1st week*

Relations, algebraic structures, operations and their properties.

*2nd week*

Peano axioms, natural numbers.

*3rd week*

Integer and rational numbers.

*4th week*

Complex numbers, operations, conjugate, absolute value.

*5th week*

Trigonometric form of complex numbers, theorem of Moivre, n^th roots of complex numbers, roots of unity.

*6th week*

Divisibility and division with remainder in Z. Greatest common divisor, Euclidean algorithm.

*7th week*

First test.

*8th week*

Euler's phi-function, the theorem of Euler-Fermat. Linear congruences. Condition of solvability, number of solutions. Linear congruence systems, Chinese remainder theorem.

*9th week*

Two-variable linear Diophantine equations, condition of solvability and their connection with linear congruences, multivariate linear Diophantine equations.

*10th week*

Polynomial ring over field. Euclidean division, greatest common divisor.

*11th week*

Ring of Z[x], Q[x], R[x], C[x], irreducible factorization.

*12th week*

Fundamental theorem of algebra. Partial fraction expression. Algebraic equations, discriminant, resultant, multiple roots, cubic and quartic equations.

*13th week*

Multivariate polynomials, symmetric and elementary symmetric functions, fundamental theorem of symmetric polynomial.

*14th week*

Second test.

**Requirements:**

*- for a signature*

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

*- for a grade*

The course is evaluated on the basis of two written tests during the semester. The grade is given according to the following table:

| Total Score (%) | Grade |
|---|---|
| 0 – 50 | fail (1) |
| 51 – 60 | pass (2) |
| 61 – 70 | satisfactory (3) |
| 71 – 85 | good (4) |
| 86 – 100 | excellent (5) |

If a student fail to pass at first attempt, then a retake of the tests is possible.

*-an offered grade:*

It is not possible to obtain an offered grade in this course.

**Person responsible for course:** Prof. Dr. Ákos Pintér, university professor, DSc

**Lecturer:** Prof. Dr. Ákos Pintér, university professor, DSc

| **Title of course**: Algebra 1.<br>**Code**: TTMBE0104 | **ECTS Credit points:** 3 |
|---|---|

**Type of teaching, contact hours**
- lecture: 2 hours/week
- practice: -
- laboratory: -

**Evaluation:** exam

**Workload (estimated), divided into contact hours:**
- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 62 hours

Total: 90 hours

**Year, semester**: 1st year, 2nd semester

**Its prerequisite(s)**: TTMBE0101

**Further courses built on it**: TTMBE0105, TTMBE0106

**Topics of course**

Definition of groups, examples. Permutations, sign of permutations. Homomorphisms. Order, cyclic groups. Subgroups, generated subgroups, Lagrange´s theorem. Direct product, the fundamental theorem of finite Abelian groups. Permutation groups and group actions, Cayley´s theorem. Homomorphisms and normal subgroups, conjugation. Factor groups. Homomorphism theorem. Isomorphism theorems. Basic properties of p-groups, center. Definition of rings, examples. Subrings, generated subrings. Finite rings without zero divisors. Homomorphisms and ideals, factor rings. Rings of polynomials. Euclidean rings and principal ideal domains, the fundamental theorem of number theory. Fields, simple algebraic extensions. Minimal polynomial. The multiplicativity formula for degrees. Algebraic numbers. Construction of the splitting field. Characteristics, prime fields. Construction of finite fields, primitive roots, subfields of finite fields. Existence of irreducible polynomials over Zp with arbitrary degree. Geometric constructions with compass and straightedge: The impossibility of doubling a cube (a. k. a. the Delian problem), trisecting an edge and squaring a circle.

**Literature**

*Compulsory:*

-

*Recommended:*

John B. Fraleigh: A first course in abstract algebra, Addison-Wesley Publishing Company, 1989.
Derek J. S. Robinson: A course in the theory of groups, Sringer-Verlag, 1980.

**Schedule:**

*1st week*

Groups: definition, basic properties, examples. Permutations, sign. Homomorphisms.

*2nd week*

Order, cyclic groups, fundamental properties.

*3rd week*

Subgroups, generated subgroups, Lagrange´s theorem.

*4th week*

Direct product, fundamental theorem of finite Abelian groups (without proof). Permutation groups and group actions, Cayley´s theorem.

*5th week*

Homomorphisms and normal subgroups, conjugation. Factor group. Homomorphism theorem.

*6th week*

Isomorphism theorems (without proof). Fundamental properties of p-groups, non-triviality of the center.

*7th week*

First test.

*8th week*

Rings: definition, basic properties, examples. Subrings, generated subrings,. Finite rings with no zero divisors are division rings.

*9th week*

Homomorphisms and ideals, factor rings and their subrings. Polynomial rings.

*10th week*

Euclidean domains and PIDs, basic number theoretical properties. Fundamental theorem of number theory in Euclidean domains.

*11th week*

Fields, simple field extensions by an algebraic element. Minimal polynomial and degree of simple extensions. Field extensions by more than one elements.

*12th week*

The multiplicativity formula for degrees. Algebraic numbers, the field of algebraic numbers is algebraically closed. Construction of the quotient field (without the proof of uniqueness).

*13th week*

Characteristic of a field, prime field. Construction of finite fields, primitive roots, subfields of finite fields. Existence of irreducible polynomials of arbitrary degree over the p-element field. Geometric constructions: the Delean problem, trisection of an angle and squaring a circle are unsolvable with a compass and a straightedge.

*14th week*

Second test.

**Requirements:**

*- for a signature*

If the student fail the course TTMBG0104, then the signature is automatically denied.

*- for a grade*

The course ends in oral examination. The grade is given according to the following table:

| **Total Score (%)** | **Grade** |
|---|---|
| 0 – 39 | fail (1) |
| 40 – 49 | pass (2) |
| 50 – 59 | satisfactory (3) |
| 60 – 69 | good (4) |
| 70 – 100 | excellent (5) |

*-an offered grade:*

It is not possible to obtain an offered grade in this course.

**Person responsible for course:** Dr. Márton Szikszai, assistant professor, PhD

| |
|---|
| **Lecturer:** Dr. Márton Szikszai, assistant professor, PhD |

| **Title of course**: Algebra 1.<br>**Code**: TTMBG0104 | **ECTS Credit points:** 2 |
|---|---|

**Type of teaching, contact hours**

- lecture: -
- practice: 2 hours/week
- laboratory: -

**Evaluation:** mid-semester grade

**Workload (estimated), divided into contact hours:**

- lecture: -
- practice: 28 hours
- laboratory: -
- home assignment: -
- preparation for the exam: 32 hours

Total: 60 hours

**Year, semester**: 1st year, 2nd semester

**Its prerequisite(s)**: TTMBE0101

**Further courses built on it**: TTMBE0105, TTMBG0105, TTMBE0106

**Topics of course**

Definition of groups, examples. Permutations, sign of permutations. Homomorphisms. Order, cyclic groups. Subgroups, generated subgroups, Lagrange´s theorem. Direct product, the fundamental theorem of finite Abelian groups. Permutation groups and group actions, Cayley´s theorem. Homomorphisms and normal subgroups, conjugation. Factor groups. Homomorphism theorem. Isomorphism theorems. Basic properties of p-groups, center. Definition of rings, examples. Subrings, generated subrings. Finite rings without zero divisors. Homomorphisms and ideals, factor rings. Rings of polynomials. Euclidean rings and principal ideal domains, the fundamental theorem of number theory. Fields, simple algebraic extensions. Minimal polynomial. The multiplicativity formula for degrees. Algebraic numbers. Construction of the splitting field. Characteristics, prime fields. Construction of finite fields, primitive roots, subfields of finite fields. Existence of irreducible polynomials over Zp with arbitrary degree. Geometric constructions with compass and straightedge: The impossibility of doubling a cube (a. k. a. the Delian problem), trisecting an edge and squaring a circle.

**Literature**

*Compulsory:*

-

*Recommended:*

John B. Fraleigh: A first course in abstract algebra, Addison-Wesley Publishing Company, 1989.

Derek J. S. Robinson: A course in the theory of groups, Sringer-Verlag, 1980.

**Schedule:**

*1st week*

Groups: definition, basic properties, examples. Permutations, sign. Homomorphisms.

*2nd week*

Order, cyclic groups, fundamental properties.

*3rd week*

Subgroups, generated subgroups, Lagrange´s theorem.

*4th week*

Direct product, fundamental theorem of finite Abelian groups (without proof). Permutation groups and group actions, Cayley´s theorem.

*5th week*

Homomorphisms and normal subgroups, conjugation. Factor group. Homomorphism theorem.

*6th week*

Isomorphism theorems (without proof). Fundamental properties of p-groups, non-triviality of the center.

*7th week*

Students can ask questions and get an overview on the subject material in all topics prior to the first test.

*8th week*

Rings: definition, basic properties, examples. Subrings, generated subrings,. Finite rings with no zero divisors are division rings.

*9th week*

Homomorphisms and ideals, factor rings and their subrings. Polynomial rings.

*10th week*

Euclidean domains and PIDs, basic number theoretical properties. Fundamental theorem of number theory in Euclidean domains.

*11th week*

Fields, simple field extensions by an algebraic element. Minimal polynomial and degree of simple extensions. Field extensions by more than one elements.

*12th week*

The multiplicativity formula for degrees. Algebraic numbers, the field of algebraic numbers is algebraically closed. Construction of the quotient field (without the proof of uniqueness).

*13th week*

Characteristic of a field, prime field. Construction of finite fields, primitive roots, subfields of finite fields. Existence of irreducible polynomials of arbitrary degree over the p-element field. Geometric constructions: the Delean problem, trisection of an angle and squaring a circle are unsolvable with a compass and a straightedge.

*14th week*

Students can ask questions and get an overview on the subject material in all topics prior to the second test.

**Requirements:**

*- for a signature*

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

*- for a grade*

The preliminary requirement to pass the course is to obtain at least 51 percent of total points from short tests written on a once a week basis with the exception of the 1st, 7th and 14th week.

The course is evaluated on the basis of two written tests during the semester. The grade is given according to the following table:

| **Total Score (%)** | **Grade** |
|---|---|
| 0 – 39 | fail (1) |
| 40 – 49 | pass (2) |

| | |
|---|---|
| 50 – 59 | satisfactory (3) |
| 60 – 69 | good (4) |
| 70 – 100 | excellent (5) |

If a student fail to pass at first attempt, then a retake of the tests is possible.
*-an offered grade:*
It is not possible to obtain an offered grade in this course.

**Person responsible for course:** Dr. Márton Szikszai, assistant professor, PhD

**Lecturer:** Dr. Márton Szikszai, assistant professor, PhD

| **Title of course**: Algebra 2.<br>**Code**: TTMBE0105 | **ECTS Credit points:** 3 |
|---|---|
| **Type of teaching, contact hours**<br>- lecture: 2 hours/week<br>- practice: -<br>- laboratory: - | |
| **Evaluation:** exam | |
| **Workload (estimated), divided into contact hours:**<br>- lecture: 28 hours<br>- practice: -<br>- laboratory: -<br>- home assignment: -<br>- preparation for the exam: 62 hours<br>Total: 90 hours | |
| **Year, semester**: 2nd year, 1st semester | |
| **Its prerequisite(s)**: TTMBE0104 | |
| **Further courses built on it**: | |

**Topics of course**

Sylow´s theorems. Semidirect product. Maximal subgroups of p-groups are normal of index p. Characteristic subgroup, commutator. Solvable groups and their basic properties. Alternating group is simple if acting on at least 5 points. Free groups, generators, relations, Dyck´s theorem. Necessary and sufficient condition for a ring to be a unique factorization domain, ascending and descending chain conditions. Field of fractions. Artinian and Noetherian rings, Hilbert´s basis theorem. Algebras, minimal polynomial over algebras. Frobenius´ theorem. Splitting field, existence, uniqueness, algebraic closure, existence. Normal extensions, extensions of perfect fields are simple. Galois theory. Funadamental theorem of algebra. Compass and straightedge constructions. Theorem of Abel and Ruffini, Casus Irreducibilis is unavoidable for degree three polynomials.

**Literature**

*Compulsory:*

-

*Recommended:*

John B. Fraleigh: A first course in abstract algebra, Addison-Wesley Publishing Company, 1989.
Derek J. S. Robinson: A course in the theory of groups, Sringer-Verlag, 1980.

**Schedule:**

*1st week*

Sylow's theorems. Semidirect products.

*2nd week*

Maximal subgroups of p-groups are normal of index p. Characteristic subgroup, commutator. Fundamental theorem of finite Abelian groups.

*3rd week*

Isomorphism theorems. Solvable groups and their basic properties. Alternating group is simple if acting on at least 5 points.

*4th week*

Free groups, generators, relations, Dyck's theorem.

*5th week*

Necessary and sufficient condition for a ring to be a unique factorization domain, ascending and descending chain conditions.

*6th week*

Field of fractions. Artinian and Noetherian rings, Hilbert's basis theorem.

*7th week*

First test.

*8th week*

Algebras, minimal polynomial over algebras, Frobenius' theorem.

*9th week*

Splitting field, existence, uniqueness, algebraic closure existence, uniqueness.

*10th week*

Normal extensions, finite extensions of perfect fields are simple.

*11th week*

Fundamental theorem of Galois theory.

*12th week*

Fundamental theorem of algebra. Compass and straightedge constructions.

*13th week*

Theorem of Abel and Ruffini, Casus Irreducibilis is unavoidable for degree three polynomials.

*14th week*

Second test.

**Requirements:**

*- for a signature*

If the student fail the course TTMBG0105, then the signature is automatically denied.

*- for a grade*

The course ends in oral examination. The grade is given according to the following table:

| Total Score (%) | Grade |
|---|---|
| 0 – 39 | fail (1) |
| 40 – 49 | pass (2) |
| 50 – 59 | satisfactory (3) |
| 60 – 69 | good (4) |
| 70 – 100 | excellent (5) |

*-an offered grade:*

It is not possible to obtain an offered grade in this course.

**Person responsible for course:** Dr. Márton Szikszai, assistant professor, PhD

**Lecturer:** Dr. Márton Szikszai, assistant professor, PhD

| **Title of course**: Algebra 2.<br>**Code**: TTMBG0105 | **ECTS Credit points:** 2 |
|---|---|

**Type of teaching, contact hours**
- lecture: -
- practice: 2 hours/week
- laboratory: -

**Evaluation:** mid-semester grade

**Workload (estimated), divided into contact hours:**
- lecture: -
- practice: 28 hours
- laboratory: -
- home assignment: -
- preparation for the exam: 32 hours

Total: 60 hours

**Year, semester**: 2nd year, 1st semester

**Its prerequisite(s)**: TTMBE0104

**Further courses built on it**: -

**Topics of course**

Sylow´s theorems. Semidirect product. Maximal subgroups of p-groups are normal of index p. Characteristic subgroup, commutator. Solvable groups and their basic properties. Alternating group is simple if acting on at least 5 points. Free groups, generators, relations, Dyck´s theorem. Necessary and sufficient condition for a ring to be a unique factorization domain, ascending and descending chain conditions. Field of fractions. Artinian and Noetherian rings, Hilbert´s basis theorem. Algebras, minimal polynomial over algebras. Frobenius´ theorem. Splitting field, existence, uniquencess, algebraic closure, existence. Normal extensions, extensions of perfect fields are simple. Galois theory. Funadamental theorem of algebra. Compass and straightedge constructions. Theorem of Abel and Ruffini, Casus Irreducibilis is unavoidable for degree three polynomials.

**Literature**

*Compulsory:*

-

*Recommended:*

John B. Fraleigh: A first course in abstract algebra, Addison-Wesley Publishing Company, 1989.

Derek J. S. Robinson: A course in the theory of groups, Sringer-Verlag, 1980.

**Schedule:**

*1st week*

Sylow's theorems. Semidirect products.

*2nd week*

Maximal subgroups of p-groups are normal of index p. Characteristic subgroup, commutator. Fundamental theorem of finite Abelian groups.

*3rd week*

Isomorphism theorems. Solvable groups and their basic properties. Alternating group is simple if acting on at least 5 points.

*4th week*

Free groups, generators, relations, Dyck's theorem.

*5th week*

Necessary and sufficient condition for a ring to be a unique factorization domain, ascending and descending chain conditions.

*6th week*

Field of fractions. Artinian and Noetherian rings, Hilbert's basis theorem.

*7th week*

Students can ask questions and get an overview on the subject material in all topics prior to the first test.

*8th week*

Algebras, minimal polynomial over algebras, Frobenius' theorem.

*9th week*

Splitting field, existence, uniqueness, algebraic closure existence, uniqueness.

*10th week*

Normal extensions, finite extensions of perfect fields are simple.

*11th week*

Fundamental theorem of Galois theory.

*12th week*

Fundamental theorem of algebra. Compass and straightedge constructions.

*13th week*

Theorem of Abel and Ruffini, Casus Irreducibilis is unavoidable for degree three polynomials.

*14th week*

Students can ask questions and get an overview on the subject material in all topics prior to the second test.

**Requirements:**

*- for a signature*

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

*- for a grade*

The preliminary requirement to pass the course is to obtain at least 51 percent of total points from short tests written on a once a week basis with the exception of the 1st, 7th and 14th week.

The course is evaluated on the basis of two written tests during the semester. The grade is given according to the following table:

| Total Score (%) | Grade |
|---|---|
| 0 – 39 | fail (1) |
| 40 – 49 | pass (2) |
| 50 – 59 | satisfactory (3) |
| 60 – 69 | good (4) |
| 70 – 100 | excellent (5) |

If a student fail to pass at first attempt, then a retake of the tests is possible.

*-an offered grade:*

It is not possible to obtain an offered grade in this course.

**Person responsible for course:** Dr. Márton Szikszai, assistant professor, PhD

| |
|---|
| **Lecturer:** Dr. Márton Szikszai, assistant professor, PhD |

| **Title of course**: Number theory<br>**Code**: TTMBE0106 | **ECTS Credit points:** 3 |
|---|---|

**Type of teaching, contact hours**
- lecture: 2 hours/week
- practice: -
- laboratory: -

**Evaluation:** exam

**Workload (estimated), divided into contact hours:**
- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 62 hours

Total: 90 hours

**Year, semester**: 2nd year, 1st semester

**Its prerequisite(s)**: TTMBE0104

**Further courses built on it**: TTMBE0109, TTMBG0110

**Topics of course**

Orders of elements, generators and their description in Zp. Quadratic residues modulo p. Residues of higher degree. Arithmetic functions. Additive and multiplicative functions, some important arithmetic functions. Summatory function and Mobius-transform of arithmetic functions. The infinitude of the set of primes. Famous problems concerning primes. Primes in arithmetic progressions, the theorem of Dirichlet. The sume of the reciprocals of primes. The $\Pi(x)$ function, the prime number theorem. Lattices, the theorems of Blichfeldt and Minkowski and their applications. The Waring problem. Pythagorean triples. Algebraic numbers, algebraic integers. The field of algebraic numbers and the ring of algebraic integers. Algebraic number fields. Degree, basis, integers and units of a number fields. Quadratic number fields and their representations in the form $Q(\sqrt{d})$.

**Literature**

*Compulsory:*

-

*Recommended:*

I. Niven, H. S. Zuckerman, H. L. Montgomery: An Introduction to the Theory of Numbers, Wiley, 1991.

K. Ireland, M. Rosen, A classical introduction to modern number theory (Second edition), Springer-Verlag.

Tom Apostol, Introduction to Analytic Number Theory, Springer-Verlag.

**Schedule:**

*1st week*

Order of an element, generators and their description in $Z_p$.

*2nd week*

Quadratic residues modulo p. Legendre- and Jacobi-symbol and their properties. Quadratic reciprocity. Congruences of higher order.

*3rd week*

Number theoretical functions. Basic properties of additive and multiplicative functions.

*4th week*

Some important number theoretical functions, main properties and explicit formulas.

*5th week*

Summation and Mobius-transform of number theoretical functions. Mobius inversion theorem and its application to multiplicative functions.

*6th week*

The infinitude of the set of primes. Famous problems concerning primes: twin primes, Mersenne-primes, Fermat-primes, Goldbach's problems.

*7th week*

Primes in arithmetic progressions, Dirichlet's theorem, special cases, the Green-Tao theorem. The divergence of the sum of the reciprocals of primes.

*8th week*

The behavior of the $\Pi(x)$ function, estimates for $\Pi(x)$, the theorem of Chebishev. The density of the sequence of primes. The prime number theorem. Consequences, the asymptotic behavior of the n-th prime. The existence of arbitrarily long intervals containing no primes.

*9th week*

Lattices in $R^n$. Bases of a lattice, unimodular transformations and unimodular matrices. The fundamental parallelepiped and the lattice determinant. Lattices as discrete subgroups of $R^n$.

*10th week*

The theorems od Blichfeldt and Minkowski, and their applications for systems of linear Diophantine inequalities.

*11th week*

The Waring problem. Representing positive integers as sums of squares and higher powers. Pithagorean triples and reduced Pithagorean triples, Fermat's equation.

*12th week*

Algebraic numbers, algebraic integers. Degree, algebraic conjugates. The defining monic polinomial of an algebraic number and its properties. The field of the algebraic numbers, the ring of the algebraic integers. Transcendental numbers.

*13th week*

Algebraic number fields. Degree, basis, ring of integers, group of units.

*14th week*

Quadratic number fields and their representation in the form $Q(\sqrt{d})$. Norm and its properties in imaginary quadratic fields. Euklidean division in the rings of Gauss integers and Euler integers. Example for a ring having no unique factorization.

**Requirements:**

*- for a signature*

If the student fail the course TTMBG0106, then the signature is automatically denied.

*- for a grade*

The course ends in oral examination. The grade is given according to the following table:

| Total Score (%) | Grade |
|---|---|
| 0 – 50 | fail (1) |
| 51 – 60 | pass (2) |
| 61 – 70 | satisfactory (3) |
| 71 – 80 | good (4) |
| 81 – 100 | excellent (5) |

*-an offered grade:*

| |
|---|
| It is not possible to obtain an offered grade in this course. |
| **Person responsible for course:** Prof. Dr. Lajos Hajdu, university professor, DSc |
| **Lecturer:** Prof. Dr. Lajos Hajdu, university professor, DSc |

| **Title of course**: Number theory<br>**Code**: TTMBG0106 | **ECTS Credit points:** 2 |
|---|---|
| **Type of teaching, contact hours**<br>- lecture: -<br>- practice: 2 hours/week<br>- laboratory: - | |
| **Evaluation:** mid-semester grade | |
| **Workload (estimated), divided into contact hours:**<br>- lecture: -<br>- practice: 28 hours<br>- laboratory: -<br>- home assignment: -<br>- preparation for the exam: 32 hours<br>Total: 60 hours | |
| **Year, semester**: 2nd year, 1st semester | |
| **Its prerequisite(s)**: TTMBE0104 | |
| **Further courses built on it**: - | |

**Topics of course**

Orders of elements, generators and their description in Zp. Quadratic residues modulo p. Residues of higher degree. Arithmetic functions. Additive and multiplicative functions, some important arithmetic functions. Summatory function and Mobius-transform of arithmetic functions. The infinitude of the set of primes. Famous pronblems concerning primes. Primes in arithmetic progressions, the theorem of Dirichlet. The sume of the reciprocals of primes. The Π(x) function, the prime number theorem. Lattices, the theorems of Blichfeldt and Minkowski and their applications. The Waring problem. Pithagorean triples. Algebraic numbers, algebraic integers. The field of algebraic numbers and the ring of algebraic integers. Algebraic number fields. Degree, basis, integers and units of a number fields. Quadratic number fields and their representations int he form Q(√d).

**Literature**

*Compulsory:*

-

*Recommended:*

I. Niven, H. S. Zuckerman, H. L. Montgomery: An Introduction to the Theory of Numbers, Wiley, 1991.

K. Ireland, M. Rosen, A classical introduction to modern number theory (Second edition), Springer-Verlag.

Tom Apostol, Introduction to Analytic Number Theory, Springer-Verlag.

**Schedule:**

*1st week*

Order of an element, generators and their description in $Z_p$.

*2nd week*

Quadratic residues modulo p. Legendre- and Jacobi-symbol and their properties. Quadratic reciprocity. Congruences of higher order.

*3rd week*

Number theoretical functions. Basic properties of additive and multiplicative functions.

*4th week*

Some important number theoretical functions, main properties and explicit formulas.

*5th week*

Summation and Mobius-transform of number theoretical functions. Mobius inversion theorem and its application to multiplicative functions.

*6th week*

The infinitude of the set of primes. Famous problems concerning primes: twin primes, Mersenne-primes, Fermat-primes, Goldbach's problems. Primes in arithmetic progressions, Dirichlet's theorem, special cases, the Green-Tao theorem.

*7th week*

First test.

*8th week*

The divergence of the sum of the reciprocals of primes. The behavior of the $\Pi(x)$ function, estimates for $\Pi(x)$, the theorem of Chebishev. The density of the sequence of primes. The prime number theorem. Consequences, the asymptotic behavior of the n-th prime. The existence of arbitrarily long intervals containing no primes.

*9th week*

Lattices in $R^n$. Bases of a lattice, unimodular transformations and unimodular matrices. The fundamental parallelepiped and the lattice determinant. Lattices as discrete subgroups of $R^n$.

*10th week*

Theorems of Minkowski and Blichfeldt and applications concerning system of linear inequalities.

*11th week*

The Waring problem. Representing positive integers as sums of squares and higher powers. Pithagorean triples and reduced Pithagorean triples, Fermat's equation.

*12th week*

Algebraic numbers, algebraic integers. Degree, algebraic conjugates. The defining monic polinomial of an algebraic number and its properties. The field of the algebraic numbers, the ring of the algebraic integers. Transcendental numbers.

*13th week*

Algebraic number fields. Degree, basis, ring of integers, group of units. Quadratic number fields and their representation in the form $Q(\sqrt{d})$. Norm and its properties in imaginary quadratic fields. Euklidean division in the rings of Gauss integers and Euler integers. Example for a ring having no unique factorization.

*14th week*

Second test.

**Requirements:**

*- for a signature*

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

*- for a grade*

The course is evaluated on the basis of two written tests during the semester. The grade is given according to the following table:

| Total Score (%) | Grade |
|---|---|
| 0 – 60 | fail (1) |
| 61 – 70 | pass (2) |

| | |
|---|---|
| 71 – 80 | satisfactory (3) |
| 81 – 90 | good (4) |
| 91 – 100 | excellent (5) |

If a student fail to pass at first attempt, then a retake of the tests is possible.
*-an offered grade:*
It is not possible to obtain an offered grade in this course.

**Person responsible for course:** Prof. Dr. Lajos Hajdu, university professor, DSc

**Lecturer:** Prof. Dr. Lajos Hajdu, university professor, DSc

| **Title of course**: Combinatorics and graph theory<br>**Code**: TTMBE0107 | **ECTS Credit points:** 4 |
|---|---|
| **Type of teaching, contact hours**<br>- lecture: 3 hours/week<br>- practice: -<br>- laboratory: - | |
| **Evaluation:** exam | |
| **Workload (estimated), divided into contact hours:**<br>- lecture: 42 hours<br>- practice: -<br>- laboratory: -<br>- home assignment: -<br>- preparation for the exam: 78 hours<br>Total: 120 hours | |
| **Year, semester**: 1st year, 1st semester | |
| **Its prerequisite(s)**: - | |
| **Further courses built on it**: TTMBE0606 | |

**Topics of course**

Fundamental enumeration problems: permutations, variations, combinations. Properties of binomial coefficients, binomial and multinomial theorem. Inversions, parity, product of permutations, cycles. Inclusion–exclusion principle and applications. Basic definitions of graph theory. Eulerian trail, Hamiltonian path and cycle. Trees and forests, spanning trees, Prüfer code and Cayley's formula. Bipartite graphs. Plane graphs, dual graph, Euler's formula, planar graphs and their characterization. Vertex and edge colourings of graphs, chromatic number, the five color theorem, chromatic polynomial, chromatic index. Fundamentals of Ramsey theory. Matrices of graphs.

**Literature**

*Compulsory:*

-

*Recommended:*

Béla Andrásfai: Introductory Graph Theory, Akadémiai Kiadó, 1977.
N. Ya. Vilenkin: Combinatorics, Academic Press, 1971.
Miklós Bóna: A Walk Through Combinatorics, World Scientific, 2017.

**Schedule:**

*1st week*

Pigeonhole principle and applications. Factorials, Stirling’s formula, binomial coefficients.

*2nd week*

Permutations, variations, combinations with and without repetitions. Properties and sums of binomial coefficients.

*3rd week*

Binomial and multinomial theorem. Inversions, parity, product of permutations, cycles.

*4th week*

Inclusion–exclusion principle and applications. Basic definitions and theorems of graph theory.

*5th week*

Graphs with given degree sequences. Walk, trail, path, cycle, connected graph, distance.

*6th week*

Eulerian trail, Hamiltonian path, Hamiltonian cycle, and theorems on their existence.

*7th week*

Trees and forests, equivalent definitions of trees. Spanning trees, spanning forests, Prüfer code, Cayley's formula.

*8th week*

Bipartite graphs and characterization theorem. Plane graphs, dual graph, Euler's formula.

*9th week*

Planar graphs, Kuratowski's theorem.

*10th week*

Vertex colourings of graphs, chromatic number and bounds. Chromatic number of planar graphs, the five and four colour theorem.

*11th week*

Chromatic polynomial and properties, chromatic polynomial of trees. Edge colourings of graphs, chromatic index and bounds.

*12th week*

Ramsey numbers: the two-colour and the multicolour case, bounds, special values.

*13th week*

Adjacency and incidence matrices of graphs, characterization of fundamental graph properties using these matrices.

*14th week*

Fundamentals of the theory of directed graphs, directed acyclic graphs.

**Requirements:**

*- for a signature*

If the student fail the course TTMBG0107, then the signature is automatically denied.

*- for a grade*

The course ends in oral examination. The grade is given according to the following table:

| Total Score (%) | Grade |
|---|---|
| 0 – 50 | fail (1) |
| 51 –60 | pass (2) |
| 61 – 70 | satisfactory (3) |
| 71 – 80 | good (4) |
| 81 – 100 | excellent (5) |

*-an offered grade:*

It is not possible to obtain an offered grade in this course.

**Person responsible for course:** Dr. Gábor Nyul, assistant professor, PhD

**Lecturer:** Dr. Gábor Nyul, assistant professor, PhD

| **Title of course**: Combinatorics and graph theory<br>**Code**: TTMBG0107 | **ECTS Credit points:** 2 |
|---|---|
| **Type of teaching, contact hours**<br>- lecture: -<br>- practice: 2 hours/week<br>- laboratory: - | |
| **Evaluation:** mid-semester grade | |
| **Workload (estimated), divided into contact hours:**<br>- lecture: -<br>- practice: 28 hours<br>- laboratory: -<br>- home assignment: -<br>- preparation for the exam: 32 hours<br>Total: 60 hours | |
| **Year, semester**: 1st year, 1st semester | |
| **Its prerequisite(s)**: - | |
| **Further courses built on it**: - | |

**Topics of course**

Fundamental enumeration problems: permutations, variations, combinations. Properties of binomial coefficients, binomial and multinomial theorem. Inversions, parity, product of permutations, cycles. Inclusion–exclusion principle and applications. Basic definitions of graph theory. Eulerian trail, Hamiltonian path and cycle. Trees and forests, spanning trees, Prüfer code and Cayley's formula. Bipartite graphs. Plane graphs, dual graph, Euler's formula, planar graphs and their characterization. Vertex and edge colourings of graphs, chromatic number, the five color theorem, chromatic polynomial, chromatic index. Fundamentals of Ramsey theory. Matrices of graphs.

**Literature**

*Compulsory:*

-

*Recommended:*

Béla Andrásfai: Introductory Graph Theory, Akadémiai Kiadó, 1977.
N. Ya. Vilenkin: Combinatorics, Academic Press, 1971.
Miklós Bóna: A Walk Through Combinatorics, World Scientific, 2017.

**Schedule:**

*1st week*

Pigeonhole principle.

*2nd week*

Elementary combinatorial exercises.

*3rd week*

Elementary combinatorial exercises.

*4th week*

Combinatorial exercises under certain restrictions.

*5th week*

Parity, product of permutations, cycles.

*6th week*

Binomial and multinomial theorem.

*7th week*

First test.

*8th week*

Inclusion–exclusion principle.

*9th week*

Graphs with given degree sequences, Havel-Hakimi theorem.

*10th week*

Walk, trail, path, cycle, connectedness, distance.

*11th week*

Eulerian trail, Hamiltonian path, Hamiltonian cycle.

*12th week*

Trees and forests, Prüfer code.

*13th week*

Adjacency and incidence matrices of graphs. Chromatic polynomial of graphs.

*14th week*

Second test.

**Requirements:**

*- for a signature*

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

*- for a grade*

The course is evaluated on the basis of two written tests during the semester. The grade is given according to the following table:

| Total Score (%) | Grade |
|---|---|
| 0 – 60 | fail (1) |
| 61 – 70 | pass (2) |
| 71 – 80 | satisfactory (3) |
| 81 – 90 | good (4) |
| 91 – 100 | excellent (5) |

If a student fail to pass at first attempt, then a retake of the tests is possible.

*-an offered grade:*

It is not possible to obtain an offered grade in this course.

**Person responsible for course:** Dr. Gábor Nyul, assistant professor, PhD

**Lecturer:** Dr. Gábor Nyul, assistant professor, PhD

**Title of course**: Sets and functions
**Code**: TTMBE0201

**ECTS Credit points:** 3

**Type of teaching, contact hours**

- lecture: 2 hours/week
- practice: -
- laboratory: -

**Evaluation:** exam

**Workload (estimated), divided into contact hours:**

- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 62 hours

Total: 90 hours

**Year, semester**: 1st year, 1st semester

**Its prerequisite(s)**: -

**Further courses built on it**: TTMBE0202, TTMBG0202

**Topics of course**

Foundations of set theory. Relations. Equivalence and order relations, functions. Basic notions in partially ordered sets and Tarski's fixed point theorem. Cardinality of sets, Cantor's theorem and the Schröder–Bernstein theorem. Axioms of the real numbers and their corollaries. Notable subsets of the reals: natural numbers, integers, rational and irrational numbers. Uniqueness of the set of real numbers. Existence and uniqueness of the nth root of a nonnegative number. The p-adic represantion of real numbers. Notable inequalities. The field of complex numbers. Cardinality of sets of numbers.

**Literature**

*Compulsory:*
- Walter Rudin: Principles of Mathematical Analysis, McGraw-Hill, New York, 1976.

**Schedule:**

1st week

Basic notions of set theory. Axiom of empty set, axiom of extensionality, axiom of pair, axiom of union, axiom of power set. Axiom of separation, Russel's theorem. Operations with sets, properties of the operations and De Morgan's laws.

2nd week

Ordered pairs, Cartesian product. Relations; domain, range and inverse of a relation. Composition of relations, properties of composition.

3rd week

The notion of a function, injective, surjective and bijective functions. Connections between functions and set operations. Indexed families of sets, axiom of choice.

4th week

Equivalence relations and partitions. Ordering relations and partial orderings, chains and intervals. Boundedness, minimum, maximum, infimum, supremum.

5th week

Completeness. Equivalent formulations of the axiom of choice: Zermelo's well ordering theorem, Hausdorff's maximum principal, Kuratowski—Zorn lemma.

6th week

Cardinality of sets. Comparison of cardinalities. Tarski's fixed point theorem and the Schröder—Bernstein theorem. Properties of relations of cardinalities.

7th week

Cardinality of a power set. Finite and infinite sets. Further axioms: axiom of regularity and axiom of infinity.

8th week

The axioms of real numbers. Corollaries of the field axioms and order axioms. The absolute value function. Dedekind's theorem and Cantor's theorem.

9th week

Natural numbers, Peano's axioms. The Archimedean property. Principle of induction and recursive definition. Properties of the binary operations. The binomial theorem and Bernoulli's inequality.

10th week

Integers, integer part and fractional part. Rational and irrational numbers, denseness theorems. Uniqueness of the set of real numbers.

11th week

Definition and existence of nth roots. Powers with rational exponents. p-adic fractions.

12th week

Notable inequalities. Power means. Inequality between the arithmetic, geometric and harmonic means. Schwarz and Minkowski inequalities.

13th week

The set of complex numbers and its algebraic structure. Real part, imaginary part, conjugate and absolute value of a complex number. Schwarz inequality for complex numbers.

14th week

Finite and infinite sets. Countable sets and the cardinality of the continuum. Cardinality of the set of natural numbers, integers, rational, real and complex numbers.

**Requirements:**

The course ends in an oral exam. The prerequisite for sitting an exam is passing the practical course. In the exam students give an account on two exam questions. Students who reveal a profound lack of knowledge will fail the exam. Students who cannot prove the theorems in their

| |
|---|
| exam questions can get at most a satisfactory (3) mark. In all other questions the Education and Examination Rules and Regulations of the University of Debrecen must be consulted. |
| **Person responsible for course:** Dr. Rezső L. Lovas, assistant professor, PhD |
| **Lecturer:** Dr. Rezső L. Lovas, assistant professor, PhD |

| **Title of course**: Sets and functions<br>**Code**: TTMBG0201 | **ECTS Credit points:** 2 |
|---|---|

**Type of teaching, contact hours**
- lecture: -
- practice: 2 hours/week
- laboratory: -

**Evaluation:** practical

**Workload (estimated), divided into contact hours:**
- lecture: -
- practice: 28 hours
- laboratory: -
- home assignment: 32 hours
- preparation for the exam: -

Total: 60 hours

**Year, semester**: 1st year, 1st semester

**Its prerequisite(s)**: -

**Further courses built on it**: TTMBE0202, TTMBG0202

**Topics of course**

Foundations of set theory. Relations. Equivalence and order relations, functions. Basic notions in partially ordered sets and Tarski's fixed point theorem. Cardinality of sets, Cantor's theorem and the Schröder–Bernstein theorem. Axioms of the real numbers and their corollaries. Notable subsets of the reals: natural numbers, integers, rational and irrational numbers. Uniqueness of the set of real numbers. Existence and uniqueness of the nth root of a nonnegative number. The p-adic represantion of real numbers. Notable inequalities. The field of complex numbers. Cardinality of sets of numbers.

**Literature**

*Compulsory:*
- Walter Rudin: Principles of Mathematical Analysis, McGraw-Hill, New York, 1976.

**Schedule:**

1st week

Operations with sets, properties of the operations and De Morgan's laws.

2nd week

Ordered pairs, Cartesian product. Relations; domain, range and inverse of a relation. Composition of relations, properties of composition.

3rd week

Special types of relations: the notion of a function, injective, surjective and bijective functions. Connections between functions and set operations.

4th week

Special types of relations: equivalence relations and partitions. Ordering relations and partial orderings. Boundedness, minimum, maximum, infimum, supremum.

$5^{th}$ week

Inequalities containing absolute values, second order polynomials and fractions of first order polynomials.

$6^{th}$ week

Further exercises and problems.

$7^{th}$ week

First mid-term test.

$8^{th}$ week

Proofs by induction.

$9^{th}$ week

Exercises to practise Cantor's theorem and the boundedness properties of natural numbers.

$10^{th}$ week

Exercises to practise notable inequalities.

$11^{th}$ week

Conversion between ordinary and p-adic fractions.

$12^{th}$ week

Problems in the arithmetic of cardinalities.

$13^{th}$ week

Further exercises and problems.

$14^{th}$ week

Second mid-term test.

**Requirements:**

Participation in practical classes is compulsory. A student must attend the practical classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. Students write to mid-term tests during the semester. At the end of the semester one of the two tests can be repeated. The result of the repeated test will replace the original one. The mark will be determined by the sum of the points of the two mid-term tests according to the following tabular:

| Score (percent) | Grade |
|---|---|
| 0—50 | fail (1) |
| 51—60 | pass (2) |
| 60—80 | satisfactory (3) |
| 81—90 | good (4) |
| 91—100 | excellent (5) |

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|---|
| In all other questions the Education and Examination Rules and Regulations of the University of Debrecen must be consulted. |
| **Person responsible for course:** Rezső L. Lovas, assistant professor, PhD |
| **Lecturer:** Rezső L. Lovas, assistant professor, PhD |

| **Title of course**: Introduction to analysis<br>**Code**: TTMBE0202 | **ECTS Credit points: 4** |
|---|---|

**Type of teaching, contact hours**

- lecture: 3 hours/week
- practice: -
- laboratory: -

**Evaluation:** exam

**Workload (estimated), divided into contact hours:**

- lecture: 42 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 78 hours

Total: 120 hours

**Year, semester**: 1st year, 2nd semester

**Its prerequisite(s)**: TTMBE0201, TTMBG0202

**Further courses built on it**: TTMBE0203

**Topics of course**

Convergence of sequences of real numbers. Relations between convergence, boundedness and monotonicity. The Bolzano–Weierstrass theorem and Cauchy's criterion for convergence. Convergence and operations, the relation between the limit and the order. Elementary sequences; the Euler number. Accumulation points, lower and upper limit of sequences. Applications. Convergence of sequences of complex numbers. The Bolzano–Weierstrass-theorem and Cauchy's criterion for sequences of complex numbers. Relations between convergence and the operations. Series of complex numbers; absolute and conditional convergence. Summation of series and operations, grouping and rearranging series. Riemann's theorem. Complex geometric series; the comparison, root and ratio tests. Abel's formula; the theorems of Dirichlet, Leibniz and Abel. Cauchy product, Mertens theorem. Pointwise and uniform convergence of function sequences and series. Cauchy's criterion and the sufficient condition of Weiersrass for uniform convergence. Power series; the Cauchy–Hadamard theorem. Elementary functions and their addition formulas. Metric spaces, normed spaces, Banach spaces, Euclidean spaces. Basic notions in metric spaces. Equivalent metrics and equivalent norms. Hausdorff's criterion for compactness. Special norms of Euclidean spaces. The Bolzano–Weierstrass-theorem and the Heine–Borel theorem. Continuity and its characterization in terms of sequences in metrc spaces. Continuity and operations, the continuity of composite functions. Relations between compactness and continuity, respectively connectedness and continuity. Continuous bijections on compact sets. Uniform continuity and its characterization.

**Literature**

*Compulsory:*

1. W. Rudin: Principles of Mathematical Analysis. McGraw-Hill, 1964.
2. E. Hewitt, K. R. Stromberg: Real and Abstract Analysis. Springer-Verlag, 1965.
3. K. R. Stromberg: An introduction to classical real analysis. Wadsworth, California, 1981.

*Recommended:*

**Schedule:**

*1st week*

Convergence of sequences of real numbers. Relations between convergence, boundedness and monotonicity. The Bolzano–Weierstrass theorem and Cauchy's criterion for convergence.

*2nd week*

Convergence and operations, the relation between the limit and the order. Elementary sequences; the Euler number.

*3rd week*

Accumulation points, lower and upper limit of sequences. Applications.

*4th week*

Convergence of sequences of complex numbers. The Bolzano–Weierstrass-theorem and Cauchy's criterion for sequences of complex numbers. Relations between convergence and the operations.

*5th week*

Complex geometric series; the comparison, root and ratio tests.

*6th week*

Abel's formula; the theorems of Dirichlet, Leibniz and Abel. Cauchy product, Mertens theorem.

*7th week*

Pointwise and uniform convergence of function sequences and series. Cauchy's criterion and the sufficient condition of Weiersrass for uniform convergence. Power series; the Cauchy–Hadamard theorem.

*8th week*

Elementary functions and their addition formulas.

*9th week*

Metric spaces, normed spaces, Banach spaces, Euclidean spaces. Basic notions in metric spaces.

*10th week*

Boundedness and uniform boundedness in metric spaces. Topology in metric spaces. Equivalent metrics and equivalent norms.

*11th week*

Compactness in metric spaces. Hausdorff's criterion for compactness.

*12th week*

Special norms of Euclidean spaces. The Bolzano–Weierstrass-theorem and the Heine–Borel theorem.

*13th week*

Continuity and its characterization in terms of sequences in metrc spaces. Continuity and operations, the continuity of composite functions.

*14th week*

Relations between compactness and continuity, respectively connectedness and continuity. Continuous bijections on compact sets. Uniform continuity and its characterization.

**Requirements:**
The course ends in an oral or written **examination**. Two assay questions are chosen randomly from the list of assays. In case one of them is incomplete, the examination ends with a fail. In lack of the knowledge of proofs, at most satisfactory can be achieved. The grade for the examination is given according to the following table:

| Score | Grade |
|---|---|
| 0-59% | fail (1) |
| 60-69% | pass (2) |
| 70-79% | satisfactory (3) |
| 80-89% | good (4) |

| |
|---|
| 90-100%     excellent (5)<br>In general, the EDUCATION AND EXAMINATION RULES AND REGULATIONS have to be taken into account. |
| **Person responsible for course:** Dr. Mihály Bessenyei, associate professor, PhD |
| **Lecturer:** Dr. Mihály Bessenyei, associate professor, PhD |

| **Title of course**: Introduction to analysis<br>**Code**: TTMBG0202 | **ECTS Credit points: 2** |
|---|---|

**Type of teaching, contact hours**

- lecture: -
- practice: 2 hours/week
- laboratory: -

**Evaluation:** mid-semester grade

**Workload (estimated), divided into contact hours:**

- lecture: -
- practice: 28 hours
- laboratory: -
- home assignment: 32 hours
- preparation for the exam: -

Total: 60 hours

**Year, semester**: 1st year, 2nd semester

**Its prerequisite(s)**: ***TTMBE0201***

**Further courses built on it**: ***TTMBE0202***

**Topics of course**

Convergence of sequences of real numbers. Convergence and operations, the relation between the limit and the order. Elementary sequences; the Euler number. Convergence of sequences of complex numbers. Series of complex numbers; absolute and conditional convergence. Summation of series and operations, grouping and rearranging series. Complex geometric series; the comparison, root and ratio tests. Pointwise and uniform convergence of function sequences and series. Cauchy's criterion and the sufficient condition of Weiersrass for uniform convergence. Power series; the Cauchy–Hadamard theorem. Elementary functions and their addition formulas. Metric spaces, normed spaces, Banach spaces, Euclidean spaces. Basic notions in metric spaces. Special norms of Euclidean spaces. Continuity and its characterization in terms of sequences in metric spaces.

**Literature**

*Compulsory:*

1. W. Rudin: Principles of Mathematical Analysis. McGraw-Hill, 1964.
2. E. Hewitt, K. R. Stromberg: Real and Abstract Analysis. Springer-Verlag, 1965.
3. K. R. Stromberg: An introduction to classical real analysis. Wadsworth, California, 1981.

*Recommended:*

**Schedule:**

*1st week*

Convergence of sequences of real numbers. Cauchy's criterion for convergence.

*2nd week*

Convergence and operations, the relation between the limit and the order. Elementary sequences; the Euler number (ratio of polynomials and exponential polynomials, difference of roots).

*3rd week*

Convergence and operations, the relation between the limit and the order. Elementary sequences; the Euler number (n-square and n-power of ratio of linear expressions).

*4th week*

Convergence of series via definition, via determining the closed form of partial sums.

*5th week*

Complex geometric series; the comparison, root and ratio tests.

*6th week*

Summary.

*7th week*

Mid-term test.

*8th week*

Power series and elementary functions.

*9th week*

Pointwise and uniform convergence of sequence and series of functions.

*10th week*

Metric spaces, normed spaces, Banach spaces, Euclidean spaces. Special norms of Euclidean spaces.

*11th week*

Topology and compactness in metric spaces.

*12th week*

Continuity and its characterization in terms of sequences in Euclidean spaces.

*13th week*

Summary.

*14th week*

End-term test.

**Requirements:**

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. A student can't make up any practice with another group. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence. In case of further absences, a medical certificate needs to be presented. Missed practice classes should be made up for at a later date, to be discussed with the tutor.

The course finishes with a grade, which is based on the total sum of points of the mid-term test (in the 7th week) and the end-term test (in the 14th week). One of the tests can be repeated. The final grade is given according to the following table:

| Score | Grade |
|---|---|
| 0-59% | fail (1) |
| 60-69% | pass (2) |
| 70-79% | satisfactory (3) |
| 80-89% | good (4) |
| 90-100% | excellent (5) |

In general, the EDUCATION AND EXAMINATION RULES AND REGULATIONS have to be taken into account.

**Person responsible for course:** Dr. Mihály Bessenyei, associate professor, PhD

**Lecturer:** Dr. Mihály Bessenyei, associate professor, PhD

| **Title of course**: Differential and integral calculus<br>**Code**: TTMBE0203 | **ECTS Credit points: 4** |
|---|---|

**Type of teaching, contact hours**
- lecture: 3 hours/week
- practice: -
- laboratory: -

**Evaluation:** exam

**Workload (estimated), divided into contact hours:**
- lecture: 42 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 78 hours

Total: 120 hours

**Year, semester**: 2nd year, 1st semester

**Its prerequisite(s)**: ***TTMBE0202, TTMBG0203***

**Further courses built on it**: ***TTMBE0204***

**Topics of course**

Limit of functions and its computation using limit of sequences. Cauchy's criterions; the relation between the limit and the operations, respectively the order. The relation between limit and uniform convergence, respectively continuity and uniform convergence; Dini's theorem. Right- and left-sided limits; points of discontinuity; classification of discontinuities of the first kind; limit properties of monotone functions. Elementary limits; the introduction of pi. Functions stemming from elementary functions. Differentiability and approximation with linear functions. Differentiability and continuity; differentiability and operations; the chain rule and the differentiability of the inverse function. Local extremum, Fermat principle. The mean value theorems of Rolle, Lagrange, Cauchy and Darboux. L'Hospital rules. Higher order differentiability; Taylor's theorem, monotonicity and differentiability, higher order conditions for extrema. Convex functions. The definition of antiderivatives; basic integrals, rules of integration. Riemann integral and criteria for integrability; properties of the integral and methods of integration. The main classes of integrable functions. Inequalities, mean value theorems for the Riemann integral. The Newton–Leibniz theorem and the properties of antiderivatives. The relation between Riemann-integrability and uniform convergence. Lebesgue's criterion. Improper Riemann integral and its criteria.

**Literature**

*Compulsory:*

1. W. Rudin: Principles of Mathematical Analysis. McGraw-Hill, 1964.
2. K. R. Stromberg: An introduction to classical real analysis. Wadsworth, California, 1981.

*Recommended:*

**Schedule:**

*1st week*

Limit of functions and its computation using limit of sequences. Cauchy's criterions; the relation between the limit and the operations, respectively the order.

*2nd week*

The relation between limit and uniform convergence, respectively continuity and uniform convergence; Dini's theorem.

*3rd week*
Right- and left-sided limits; points of discontinuity; classification of discontinuities of the first kind; limit properties of monotone functions.

*4th week*
Elementary limits; the introduction of pi. Functions stemming from elementary functions.

*5th week*
Differentiability and approximation with linear functions. Differentiability and continuity; differentiability and operations; the chain rule and the differentiability of the inverse function.

*6th week*
Local extremum, Fermat principle. The mean value theorems of Rolle, Lagrange, Cauchy and Darboux. L'Hospital rules. Higher order differentiability; Taylor's theorem.

*7th week*
Monotonicity and differentiability, higher order conditions for extrema. Convex functions.

*8th week*
The definition of antiderivatives; basic integrals, rules of integration.

*9th week*
Darboux integrals and their properties.

*10th week*
Riemann integral and its properties.

*11th week*
The main classes of integrable functions. Inequalities, mean value theorems for the Riemann integral. The Newton–Leibniz theorem and the properties of antiderivatives.

*12th week*
The relation between Riemann-integrability and uniform convergence. Applications. Improper Riemann-integral.

*13th week*
Lebesgue null sets. Modulus of continuity.

*14th week*
Lebesgue's criterion and its applications.

**Requirements:**
The course ends in an oral or written **examination**. Two assay questions are chosen randomly from the list of assays. In case one of them is incomplete, the examination ends with a fail. In lack of the knowledge of proofs, at most satisfactory can be achieved. The grade for the examination is given according to the following table:

| Score | Grade |
|---|---|
| 0-59% | fail (1) |
| 60-69% | pass (2) |
| 70-79% | satisfactory (3) |
| 80-89% | good (4) |
| 90-100% | excellent (5) |

In general, the EDUCATION AND EXAMINATION RULES AND REGULATIONS have to be taken into account.

**Person responsible for course:** Dr. Mihály Bessenyei, associate professor, PhD

**Lecturer:** Dr. Mihály Bessenyei, associate professor, PhD

| **Title of course**: Differential and integral calculus<br>**Code**: TTMBG0203 | **ECTS Credit points: 3** |
|---|---|

**Type of teaching, contact hours**
- lecture:
- practice: 3 hours/week
- laboratory: -

**Evaluation:** mid-semester grade

**Workload (estimated), divided into contact hours:**
- lecture: -
- practice: 42 hours
- laboratory: -
- home assignment: 48 hours
- preparation for the exam: -

Total: 90 hours

**Year, semester**: 2nd year, 1st semester

**Its prerequisite(s)**: ***TTMBE0202***

**Further courses built on it**: ***TTMBE0203***

**Topics of course**

Limit of functions and its computation using limit of sequences. Differentiability and operations; the chain rule and the differentiability of the inverse function. Local extremum, Fermat principle, mean value theorems. L'Hospital rules. Higher order differentiability; Taylor's theorem. Monotonicity, convexity, extrema. Basic integrals, rules of integration. Riemann integral and the Newton–Leibniz theorem. Inequalities for Riemann integral. Improper Riemann integral.

**Literature**

*Compulsory:*

1. W. Rudin: Principles of Mathematical Analysis. McGraw-Hill, 1964.
2. K. R. Stromberg: An introduction to classical real analysis. Wadsworth, California, 1981.

*Recommended:*

**Schedule:**

*1st week*
Computing limits and derivatives of functions and its computation using limit of sequences.

*2nd week*
Differentiability and operations; the chain rule and the differentiability of the inverse function.

*3rd week*
Higher order differentiability; Taylor's theorem.

*4th week*
The mean value theorems of Rolle, Lagrange, Cauchy and Darboux. L'Hospital rules.

*5th week*
Monotonicity, convexity, extrema of functions.

*6th week*
Summary

*7th week*
Midterm test.

*8th week*
Basic integrals, rules of integration.

*9th week*

Integration of partial fractions.

*10th week*

Applications of the integration of partial fractions.

*11th week*
Riemann sums and Riemann integral. The Newton–Leibniz theorem. Improper Riemann integrals.

*12th week*
Inequalities for Riemann integral.

*13th week*

Summary.

*14th week*
Endterm test.

**Requirements:**
Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. A student can't make up any practice with another group. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence. In case of further absences, a medical certificate needs to be presented. Missed practice classes should be made up for at a later date, to be discussed with the tutor.

The course finishes with a grade, which is based on the total sum of points of the mid-term test (in the 7th week) and the end-term test (in the 14th week). One of the tests can be repeated. The final grade is given according to the following table:

| Score | Grade |
|---|---|
| 0-59% | fail (1) |
| 60-69% | pass (2) |
| 70-79% | satisfactory (3) |
| 80-89% | good (4) |
| 90-100% | excellent (5) |

In general, the EDUCATION AND EXAMINATION RULES AND REGULATIONS have to be taken into account.

**Person responsible for course:** Dr. Mihály Bessenyei, associate professor, PhD

**Lecturer:** Dr. Mihály Bessenyei, associate professor, PhD

| **Title of course**: Differential and integral calculus in several variables<br>**Code**: TTMBE0204-EN | **ECTS Credit points: 4** |
|---|---|

**Type of teaching, contact hours**
- lecture: 3 hours/week
- practice: -
- laboratory: -

**Evaluation:** exam

**Workload (estimated), divided into contact hours:**
- lecture: 42 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 78 hours

Total: 120 hours

**Year, semester**: 2nd year, 2nd semester

**Its prerequisite(s)**: TTMBE0203

**Further courses built on it**:

**Topics of course**

The Banach contraction principle. Linear maps in normed spaces. The Fréchet derivative; chain rule, differentiability and operations. The mean value inequality of Lagrange. Inverse and implicit function theorems. Further notions of derivatives; the representation of the Fréchet derivative. Continuous differentiability and continuous partial differentiability; sufficient condition for differentiability. Higher order derivatives; Schwarz–Young theorem, Taylor's theorem. Local extremum and Fermat principle; the second order conditions for extrema. The Lagrange Multiplier Rule. The definition of the Riemann integral; the integral and operations, criteria for integrability, inequalities and mean value theorems for the Riemann integral. The relation between the Riemann integral and the uniform convergence. Lebesgue's theorem. Fubini's theorem. Jordan measure and its properties; integration over Jordan measurable sets. Fubini's theorem on simple regions, integral transformation. Functions of bounded variation, total variation, decomposition theorem of Jordan. The Riemann–Stieltjes integral and its properties. Integration by parts. Sufficient condition for Riemann–Stieltjes integrability and the computation of the integral. Curve integral; potential function and antiderivative. Necessary and sufficient conditions for the existence of antiderivatives.

**Literature**

*Compulsory:-*
*Recommended:*
W. Rudin: Principles of Mathematical Analysis. McGraw-Hill, 1964.
K. R. Stromberg: An introduction to classical real analysis. Wadsworth, California, 1981.

**Schedule:**

*1st week* Metric spaces. Limit of sequences and completeness. The Banach fixed point theorem. Characterization of Banach spaces among normed spaces. Compactness in normed spaces. The equivalence of the norms in finite dimensional normed spaces. Examples.

*2nd week* The norm of linear mappings, characterizations of bounded linear maps. The structure of the space of linear maps. Convergence of Neumann series. The topological structure of invertible linear self-maps in a Banach space. The open mapping theorem and its consequences.

*3rd week* The notion of Fréchet derivative and its uniqueness. The connection of differentiability and continuity. The Fréchet derivative of affine and bilinear maps. The Chain Rule and its consequences.

*4th week* The Hahn-Banach theorem for normed spaces and the Lagrange mean value inequality. Strict and continuous Fréchet differentiability. The inverse and implicit function theorems.

*5th week* The notions of directional and partial derivatives and their connection to Fréchet differentiability. The representation of the Fréchet derivative via partial derivatives. Sufficient condition for Fréchet differentiability, the characterization of continuous differentiability.

*6th week* Higher-order derivatives, the Schwarz-Young theorem and the Taylor theorem. Local minimum and maximum, the Fermat principle. Characterizations of positive definite and positive semidefinite quadratic forms. The second-order necessary and sufficient conditions of optimality. Constrained optimization and the Lagrange multiplier rule.

*7th week* Compact intervals in Euclidean spaces. Subdivision of intervals. The lower and upper integral approximating sums of bounded functions and their basic properties. The lower and upper Darboux integrals and their properties. The Darboux theorem. The interval additivity of the Darboux integrals.

*8th week* The notion of the Riemann integral and examples for non-integrability. The linearity and interval additivity of the Riemann integral. The Riemann criterion of integrability. Further criteria of integrability.

*9th week* Integrability and continuity. Sufficient conditions of integrability. Operations with Riemann integrable functions. Mean value theorem for the Riemann integral. Uniform convergence and integrability. The structure of the space of Riemann integrable functions.

*10th week* Computation of the Riemann integral, the Fubini theorem and its consequences. Null sets in the sense of Lebesgue and their properties. The characterization of Riemann integrability via the Lebesgue criterion.

*11th week* The Jordan measure and its properties. Characterization of Jordan measurability and Jordan null sets. The Riemann integral over Jordan measurable sets. Algebraic properties, connection integrability and continuity. The Fubini theorem on normal domains. The integral transformation theorem.

*12th week* Functions of bounded variations and their structure. The interval additivity if total variation and the Jordan decomposition theorem and its corollaries. The computation of the total variation.

*13th week* The Riemann-Stieltjes integral, its bilinearity and interval additivity. Integration by parts. Sufficient conditions for Riemann-Stieltjes integrability and the computation of the integral.

*14th week* Curves and the length of curves. The curve integral of vector fields. Antiderivative function (potential function) of vector fields. The Newton-Leibniz theorem. Differentiation of parametric integrals. The necessary and sufficient conditions for the existence of antiderivative function.

**Requirements:**

*- for a signature*

Attendance at **lectures** is recommended, but not compulsory.

*- for a grade*

The course ends in an **examination**. Before the examination students must have grade at least ‘pass’ on *Differential and integral calculus in several variables* practice (TTMBG0204-EN). The grade for the examination is given according to the following table:

| Score | Grade |
|---|---|
| 0-49 | fail (1) |
| 50-61 | pass (2) |
| 62-74 | satisfactory (3) |

| | |
|---|---|
| 75-87 | good (4) |
| 88-100 | excellent (5) |

If the average of the score of the examination is below 50, students can take a retake examination in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

**Person responsible for course:** Prof. Dr. Zsolt Páles, university professor, DSc

**Lecturer:** Prof. Dr. Zsolt Páles, university professor, DSc

| **Title of course**: Differential and integral calculus in several variables<br>**Code**: TTMBG0204-EN | **ECTS Credit points: 3** |
|---|---|

**Type of teaching, contact hours**
- lecture: -
- practice: 3 hours/week
- laboratory: -

**Evaluation:** mid-semester grade

**Workload (estimated), divided into contact hours:**
- lecture: -
- practice: 42 hours
- laboratory: -
- home assignment: 24 hours
- preparation for the tests: 24 hours

Total: 90 hours

**Year, semester**: 2nd year, 2nd semester

**Its prerequisite(s)**: TTMBE0203

**Further courses built on it**:

**Topics of course**

The Fréchet derivative, directional derivative, partial derivative. Examples for differentiability and non-differentiability. Computation of the derivatives, chain rule. The inverse and implicit function theorems. Further notions of differentiability, the representation of the Fréchet derivative. Higher order derivatives; Schwarz–Young theorem, Taylor's theorem. Local extremum and Fermat principle; the second-order conditions for extrema. The Lagrange Multiplier Rule. The computation of the Riemann integral; the integral and operations, criteria for integrability. Fubini's theorem. Jordan measure and its properties; integration over Jordan measurable sets. Fubini's theorem on simple regions, integral transformation. Functions of bounded variation, total variation. The Riemann–Stieltjes integral, integration by parts. The computation of the integral. Curve integral; potential function and antiderivative.

**Literature**

*Compulsory:-*
*Recommended:*

W. Rudin: Principles of Mathematical Analysis. McGraw-Hill, 1964.
K. R. Stromberg: An introduction to classical real analysis. Wadsworth, California, 1981.

**Schedule:**

*1st week* Limit of vector-valued functions in several variables. Checking Fréchet differentiability, directional differentiability, partial differentiability by definition.

*2nd week* The representation of the derivative in terms of partial derivatives. Computation of the directional and partial derivatives. Applications of the Chain Rule.

*3rd week* The inverse and implicit function theorems, implicit differentiation. Higher-order derivatives and differentials. Applications of the Taylor theorem.

*4th week* The Fermat principle for local minimum and maximum. Characterization of positive definite and positive semidefinite quadratic forms. The second-order necessary and sufficient conditions of optimality.

*5th week* Optimization problems with equality and inequality constraints and applications of the Lagrange multiplier rule.

*6th week* Survey of the results and methods of the first 5 weeks.

*7th week* Mid-term test.

*8th week* Computation of the Riemann-integral with the help of the Fubini theorem. The Jordan measure of bounded sets.

*9th week* Computation of the Riemann-integral with the help of the integral transformation theorem.

*10th week* Functions of bounded and of unbounded variations. The computation of total variation.

*11th week* The Riemann-Stieltjes integral and the curve integral.

*12th week* Existence and non-existence of the primitive function (potential function) of vector fields.

*13th week* Survey of the results and methods of the 8th-12th weeks.

*14th week* End-term test.

**Requirements:**

*- for a signature*

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. A student can't make up any practice with another group. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence. In case of further absences, a medical certificate needs to be presented. Missed practice classes should be made up for at a later date, to be discussed with the tutor. Active participation is evaluated by the teacher in every class. If a student's behaviour or conduct doesn't meet the requirements of active participation, the teacher may evaluate his/her participation as an absence because of the lack of active participation in class.

During the semester there are two tests: the mid-term test in the 7th week and the end-term test in the 14th week. Students have to sit for the tests.

*- for a grade*

The minimum requirement for the average of the mid-term and end-term tests is 50%.

| Score | Grade |
|---|---|
| 0-49 | fail (1) |
| 50-61 | pass (2) |
| 62-74 | satisfactory (3) |
| 75-87 | good (4) |
| 88-100 | excellent (5) |

If the average of the scores of the tests is below 50, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

**Person responsible for course:** Prof. Dr. Zsolt Páles, university professor, DSc

**Lecturer:** Prof. Dr. Zsolt Páles, university professor, DSc

| **Title of course**: Ordinary differential equations<br>**Code**: TTMBE0206-EN | **ECTS Credit points:** 3 |
|---|---|

**Type of teaching, contact hours**

- lecture: 2 hours/week
- practice: -
- laboratory: -

**Evaluation:** exam

**Workload (estimated), divided into contact hours:**

- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 62 hours

Total: 90 hours

**Year, semester**: 3rd year, 1st semester

**Its prerequisite(s)**: Differential and integral calculus in several variables: TTMBE0204

**Further courses built on it**:

**Topics of course**

Differential equations solvable in an elementary way. Cauchy problem; solution, maximal solution, locally and globally unique solution. Lipschitz condition; the theorem on global-local existence and uniqueness. Continuous dependence on the initial value. The Arzelà–Ascoli theorem and Peano's theorem. First order linear systems of differential equations; fundamental matrix, Liouville's formula, variation of constants. The construction of fundamental matrices of linear systems of differential equations with constant coefficients. Higher order (linear) differential equations and the Transition Principle; Wronski determinant and Liouville's formula. Fundamental sets of solutions of higher order linear differential equations with constant coefficients. Stability; Gronwall–Bellmann lemma and the stability theorem of Lyapunov. Elements of calculus of variations: the Du Bois-Reymond lemma and the Euler–Lagrange equations. Applications.

**Literature**

*Compulsory/Recommended:*

E. A. Coddington, N. Levinson: Theory of Ordinary Differential Equations. McGraw-Hill, 1955.

**Schedule:**

*1st week*

Ordinary explicit differential equations of first order solvable in an elementary way.

Separable, linear and exact equations. The Euler multiplicator.

*2nd week*

The notion of the Cauchy problem with respect to ordinary explicit differential equation systems of first order. Solution, complete solution, unique solution. Sufficient condition for the existence of the complete solution, global and local solvability.

*3rd week*

Complete metric spaces. The parametric version of the Banach fixed-point theorem. Weighted function spaces; The Cauchy problem and its equivalent integral equation.

*4th week*

Lipschitz properties. Global existence and uniqueness theorem. Continuous dependence on initial value; local existence and uniqueness theorem.

*5th week*

Compact operators; Schauder's fixed point theorems. Compact subsets of the space of continuous functions on intervals. Equicontinuity and uniform boundedness. Arzelà–Ascoli theorem.

*6th week*

Peano's existence theorem.

*7th week*

Linear differential equation systems of first order and their existence and uniqueness. Fundamental system and fundamental matrix; Liouville's formula. The method of constant variation.

*8th week*

The general theory of linear differential equation systems with constant coefficients: spectral radius, expression of analytic functions of matrices, the fundamental system of linear differential equation systems of first order with constant coefficient.

*9th week*

The general theory of explicit differential equations of higher order: transmission principle, Global existence and uniqueness theorem. Cauchy problem for higher order linear differential equations. The concept and the existence of the fundamental system; Wronski-determinant and Liouville formula.

*10th week*

Equivalent characterization of the fundamental system of a higher order linear linear differential equation. The constant variation method. The fundamental system of higher order homogeneous linear differential equations with constant coefficients.

*11th week*

Elements of stability theory. Definition of unstable, stable and asymptotically stable solution. Stability of the null-solution of homogeneous linear differential equation systems with constant coefficients.

*12th week*

The Gronwall–Bellmann lemma and the stability theorem of Lyapunov.

*13th week*

Elements of calculus of variation. The set of admissible functions and its topology. The differentiation of the perturbed basic functional and the Du-Bois-Reymond lemma.

*14th week*

The Euler-Lagrange differential equations. Applications: the problem of minimal surface solid of revolution, the Poincaré half-circle model of Bolyai–Lobachevsky's geometry. The Lagrange discussion of classical mechanics.

**Requirements:**

*- for a signature*
Attendance at **lectures** is recommended, but not compulsory.

*- for a grade*
The course ends in an **examination**. Before the examination students must have grade at least 'pass' on ordinary differential equations practice (TTMBG0206-EN).

The grade for the examination is given according to the following table:

| Score | Grade |
|---|---|
| 0-49 | fail (1) |
| 50-61 | pass (2) |
| 62-74 | satisfactory (3) |
| 75-87 | good (4) |
| 88-100 | excellent (5) |

If the average of the score of the examination is below 50, students can take a retake examination in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

**Person responsible for course:** Prof. Dr. György Gát, university professor, DSc

**Lecturer:** Prof. Dr. György Gát, university professor, DSc

| **Title of course**: Ordinary differential equations<br>**Code**: TTMBG0206-EN | **ECTS Credit points:** 2 |
|---|---|

**Type of teaching, contact hours**
- lecture: -
- practice: 2 hours/week
- laboratory: -

**Evaluation:** exam

**Workload (estimated), divided into contact hours:**
- lecture: -
- practice: 28 hours
- laboratory: -
- home assignment: 14 hours
- preparation for the tests: 18 hours

Total: 60 hours

**Year, semester**: 3nd year, 1st semester

**Its prerequisite(s)**: Differential and integral calculus in several variables: TTMBE0204

**Further courses built on it**:

**Topics of course**

Differential equations solvable in an elementary way.
Linear differential equation systems of first order; fundamental matrix, Liouville formula, constant variation. Construction of the fundamental matrix of linear differential equation systems with constant coefficients. Higher order (linear) differential equations and transmission principles; Wronski determinant and Liouville formula. Fundamental system of linear differential equations with constant coefficients. Elements of calculus variation: Du Bois-Reymond lemma and Euler-Lagrange equation.

**Literature**

*Compulsory/Recommended:*
E. A. Coddington, N. Levinson: Theory of Ordinary Differential Equations. McGraw-Hill, 1955.

**Schedule:**

*1st week*
Differential equations solvable in an elementary way. Separable equations.

*2nd week*
Differential equations of type that can be traced back into a separable equation (linear substitution, homogeneous equations).

*3rd week*
Types that can be traced back into a separable equation (linear fractional substitution).

*4th week*
Differential equations that can be solved in an elementary way: first order linear equations. Bernoulli and Riccati equations.

*5th week*

Differential equations that can be solved in an elementary way: exact equations, Euler's multipliers.

*6th week*

Summarize, practice and deepen the foregoing.

*7th week*

Test

*8th week*

First order homogeneous linear differential equation systems with constant coefficients. Construction of the fundamental system. Expression of analytic functions of matrices.

*9th week*

First order inhomogeneous linear differential equation systems with constant coefficient. The constant variation method

*10th week*

Higher order linear equations with constant coefficients. Transmission principle, Characteristic polynomial, reduced constant variation, test function.

*11th week*

Higher linear linear equations with variable coefficients. Wronski determinant, Liouville formula and D'Alembert reduction.

*12th week*

Elements of calculus of variation. The Euler-Lagrange differential equations.

*13th week*

Summarize, practice and deepen the foregoing.

*14th week*

Test

**Requirements:**

*- for a signature*

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. A student can't make up any practice with another group. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence. In case of further absences, a medical certificate needs to be presented. Missed practice classes should be made up for at a later date, to be discussed with the tutor. Active participation is evaluated by the teacher in every class. If a student's behaviour or conduct doesn't meet the requirements of active participation, the teacher may evaluate his/her participation as an absence because of the lack of active participation in class.

During the semester there are two tests: the mid-term test in the 7th week and the end-term test in the 14th week. Students have to sit for the tests.

*- for a grade*

The minimum requirement for the average of the mid-term and end-term tests is 50%. The score is the average of the scores of the two tests and the grade is given according to the following table:

| Score | Grade |
|---|---|
| 0-49 | fail (1) |
| 50-61 | pass (2) |
| 62-74 | satisfactory (3) |
| 75-87 | good (4) |
| 88-100 | excellent (5) |

If the average of the scores is below 50, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

**Person responsible for course:** Prof. Dr. György Gát, university professor, DSc

**Lecturer:** Prof. Dr. György Gát, university professor, DSc

| **Title of course**: Geometry 1.<br>**Code**: TTMBE0301 | **ECTS Credit points: 3** |
|---|---|

**Type of teaching, contact hours**
- lecture: 2 hours/week
- practice: -
- laboratory: -

**Evaluation:** exam

**Workload (estimated), divided into contact hours:**
- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 62 hours

Total: 90 hours

**Year, semester**: 1st year, 1st semester

**Its prerequisite(s)**: TTMBG0301 (p)

**Further courses built on it**: Geometry 2.

**Topics of course**

Absolute Geometry: incidence axioms, ruler postulate, plane separation postulate, protractor postulate and the axiom of congruence. Some representative results in Absolute Geometry: congruence theorems, perpendicular and parallel lines, sufficient conditions for parallelism, inequalities. The Euclidean parallel postulate and some equivalent statements. Introduction to the Euclidean geometry (theorems for parallelograms, Intercept theorem and its relatives, similar triangles). Euclidean plane isometries: three mirrors suffice, the classification theorem. The classification of the Euclidean space isometries. Similarities, the fixpoint theorem and the classification of plane/space similarities. The general notion of congruence and similarity. Geometric measure theory: area of polygons, Jordan measure, the area of a circle. The axioms of measuring volumes, the volume of a sphere . The perimeter of a circle, the area of a sphere.

**Literature**

Compulsory/Recommended Readings:

Csaba Vincze and László Kozma: College Geometry, TÁMOP-4.1.2.A/1-11/1-2011-0098, http://www.tankonyvtar.hu/hu/tartalom/tamop412A/2011-0098_college_geometry/index.html

John Roe: Elementary Geometry, Oxford University Press, 1993.

**Schedule:**

*1st week*

Incidence axioms.

*2nd week*

Ruler postulate, plane separation postulate.

*3rd week*

Protractor postulate and the axiom of congruence.

*4th week*

Some representative results in Absolute Geometry: congruence theorems.

*5th week*

Some representative results in Absolute Geometry: perpendicular and parallel lines, sufficient conditions for parallelism.

*6th week*

Inequalities.

*7th week*

The Euclidean parallel postulate and some equivalent statements.

*8th week*

Introduction to the Euclidean geometry (theorems for parallelograms, Intercept theorem and its relatives, similar triangles).

*9th week*

Euclidean plane isometries: three mirrors suffice, the classification theorem.

*10th week*

The classification of the Euclidean space isometries.

*11th week*

Similarities, the fixpoint theorem and the classification of plane/space similarities. The general notion of congruence and similarity.

*12th week*

Geometric measure theory: area of polygons, Jordan measure, the area of a circle.

*13th week*

The axioms of measuring volumes, the volume of a sphere.

*14th week*

The perimeter of a circle, the area of a sphere.

**Requirements:**

*- for a signature*

*- for a grade*

Attendance at **lectures** is recommended, but not compulsory. The course ends in an **examination**. A practice grade at least 2 (pass) is a prerequisite to take an exam. The minimum requirement of the exam is more than 60 %. The grade is given according to the following table:

| Percent | Grade |
|---|---|
| 0-60 | fail (1) |
| 61-70 | pass (2) |
| 71-80 | satisfactory (3) |
| 81-90 | good (4) |
| 91-100 | excellent (5) |

*-an offered grade:*

**Person responsible for course:** Dr. Csaba Vincze, associate professor, PhD

**Lecturer:** Dr. Csaba Vincze, associate professor, PhD

| **Title of course**: Geometry 1.<br>**Code**: TTMBG0301 | **ECTS Credit points:** 2 |
|---|---|

**Type of teaching, contact hours**
- lecture:
- practice: 2 hours/week
- laboratory: -

**Evaluation:** mid-semester grade

**Workload (estimated), divided into contact hours:**
- lecture:
- practice: 28 hours
- laboratory: -
- home assignment: -
- preparation for the exam: 32 hours

Total: 60 hours

**Year, semester**: 1st year, 1st semester

**Its prerequisite(s)**: -

**Further courses built on it**: Geometry 2.

**Topics of course**

Triangles and circles. Trigonometry and its applications (inaccessible distances, visibility angles). Coordinate geometry and its applications (triangles and circles), intersections. Ruler-and-compass constructions. Inversive geometry, Mohr-Masceroni's theorem. The problem of Apollonius. Conics and the reflective properties (tangent lines to ellipses, paraboles and hyperboles). Ruler-and-compass constructions related to conics. The geometry of the space (area, volume), revolution surfaces. Conic sections. The sphere (longitude and latitude), mappings of the sphere to the plane.

**Literature**

Compulsory/Recommended Readings:

Csaba Vincze and László Kozma: College Geometry, TÁMOP-4.1.2.A/1-11/1-2011-0098, http://www.tankonyvtar.hu/hu/tartalom/tamop412A/2011-0098_college_geometry/index.html

John Roe: Elementary Geometry, Oxford University Press, 1993.

**Schedule:**

*1st week*

Triangles.

*2nd week*

Circles.

*3rd week*

Trigonometry and its applications (inaccessible distances, visibility angles).

*4th week*

Coordinate geometry and its applications (triangles).

*5th week*

Coordinate geometry and its applications (circles).

*6th week*

Intersections.

*7th week*

Ruler-and-compass constructions.

*8th week*

Inversive geometry.

*9th week*

Mohr-Masceroni's theorem.

*10th week*

The problem of Apollonius.

*11th week*

Conics and the reflective properties (tangent lines to ellipses, paraboles and hyperboles). Ruler-and-compass constructions.

*12th week*

The geometry of the space (area, volume).

*13th week*

Revolution surfaces. Conic sections.

*14th week*

The sphere (longitude and latitude), mappings of the sphere to the plane.

**Requirements:**

*- for a signature*

Participation at practice classes is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence.

*- for a grade*

During the semester there are two tests. The minimum requirement for a grade is to get more than 60% of the total score of the two tests, separately. The grade is given according to the following table:

| Percent | Grade |
|---|---|
| 0-60 | fail (1) |
| 61-70 | pass (2) |
| 71-80 | satisfactory (3) |
| 81-90 | good (4) |
| 91-100 | excellent (5) |

If the score of any test is below 60%, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS. The final grade is the average of the grades of the two tests by the usual rules of rounding.

*-an offered grade:* -

**Person responsible for course:** Dr. Csaba Vincze, associate professor, PhD

**Lecturer:** Dr. Csaba Vincze, associate professor, PhD

| **Title of course**: Geometry 2.<br>**Code**: TTMBE0302 | **ECTS Credit points: 3** |
|---|---|

**Type of teaching, contact hours**
- lecture: 2 hours/week
- practice: -
- laboratory: -

**Evaluation:** exam

**Workload (estimated), divided into contact hours:**
- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 62 hours

Total: 90 hours

**Year, semester**: 1st year, 2nd semester

**Its prerequisite(s)**: TTMBE0102, TTMBG0302 (p)

**Further courses built on it**: Differential geometry

**Topics of course**

Euclidean-Affin Geometry: vectors. Affine transformations, translations and central similarities. The ratio of three collinear points. Some representative results in Affine Geometry: Ceva's theorem, Menelaus' theorem. Analytic Euclidean-Affine geometry. Linear transformations, the general linear group. The analytic description of affine transformations. The fundamental theorem. Dot and cross product, vector triple product: the geometric characterization and the analytic formulas. Higher dimensional analytic geometry: reflections and isometries of the n-dimensional Euclidean space. The orthogonal group. Lower dimensional cases: two- and three-dimesnional spaces. Coordinate geometry: lines and planes. Implicite and parametric forms. Quadratic curves and surfaces. An introduction to Convex Geometry: convex sets and convex hulls. Carathéodory's theorem. Radon's lemma and Helly's theorem. Convex polygons and polyhedra. Euler's theorem, Descartes' theorem, regular convex polyhedra.

**Literature**

Compulsory/Recommended Readings:

S. R. Lay: Convex Sets and Their Applications, John Wiley & Sons, Inc., 1982.
John Roe: Elementary Geometry, Oxford University Press, 1993.
Csaba Vincze: Convex Geometry, TÁMOP-4.1.2.A/1-11/1-2011-0025,
http://www.tankonyvtar.hu/hu/tartalom/tamop412A/2011_0025_mat_14/index.html

**Schedule:**

*1st week*

Euclidean-Affin Geometry: vectors.

*2nd week*

Affine transformations, translations and central similarities.

*3rd week*

The ratio of three collinear points. Some representative results in Affine Geometry: Ceva's theorem, Menelaus' theorem.

*4th week*

Analytic Euclidean-Affine geometry. Linear transformations, the general linear group.

*5th week*

The analytic description of affine transformations. The fundamental theorem.

*6th week*

Dot and cross product: the geometric characterization and the analytic formulas.

*7th week*

Vector triple product: the geometric characterization and the analytic formula.

*8th week*

Higher dimensional analytic geometry: reflections and isometries of the n-dimensional Euclidean space.

*9th week*

The orthogonal group.

*10th week*

Lower dimensional cases: two- and three-dimesnional spaces.

*11th week*

Coordinate geometry: lines and planes. Implicite and parametric forms.

*12th week*

Quadratic curves and surfaces.

*13th week*

An introduction to Convex Geometry: convex sets and convex hulls. Carathéodory's theorem. Radon's lemma and Helly's theorem.

*14th week*

Convex polygons and polyhedra. Euler's theorem, Descartes' theorem, regular convex polyhedra

**Requirements:**

*- for a signature*

*- for a grade*

Attendance at **lectures** is recommended, but not compulsory. The course ends in an **examination**. A practice grade at least 2 (pass) is a prerequisite to take an exam. The minimum requirement of the exam is more than 60 %. The grade is given according to the following table:

| Percent | Grade |
|---|---|
| 0-60 | fail (1) |
| 61-70 | pass (2) |
| 71-80 | satisfactory (3) |
| 81-90 | good (4) |
| 91-100 | excellent (5) |

*-an offered grade:*

**Person responsible for course:** Dr. Csaba Vincze, associate professor, PhD

**Lecturer:** Dr. Csaba Vincze, associate professor, PhD

| **Title of course**: Geometry 2.<br>**Code**: TTMBG0302 | **ECTS Credit points: 2** |
|---|---|
| **Type of teaching, contact hours**<br>- lecture:<br>- practice: 2 hours/week<br>- laboratory: - | |
| **Evaluation:** mid-semester grade | |
| **Workload (estimated), divided into contact hours:**<br>- lecture: 28 hours<br>- practice: -<br>- laboratory: -<br>- home assignment: -<br>- preparation for the exam: 32 hours<br>Total: 60 hours | |
| **Year, semester**: 1st year, 2nd semester | |
| **Its prerequisite(s)**: TTMEG0301 | |
| **Further courses built on it**: Differential geometry | |

**Topics of course**

The solution of geometric problems by vector algebra. The barycenter of a triangle and a tetrahedron. Linear dependency and independency, basis, coordinates. The simple ratio. The ellipse as the affine image of a circle. The area of an ellipse, compass-and-ruler constructions and the coordinate geometry of conics. Scalar, vector and mixed products. Computations in higher dimensional Euclidean vector spaces. Hyperplanes and reflections (analytic description). Reflections about lines and planes, rotations around lines and points. Lines, circles, planes and spheres. Intersections, distance and angles. Curves and surfaces (ruled- and revolution surfaces) implicit forms and parametrizations. Rolling without slipping: the cycloid. Twisted surfaces. Convex geometry.

**Literature**

Compulsory/Recommended Readings:

Csaba Vincze and László Kozma: College Geometry, TÁMOP-4.1.2.A/1-11/1-2011-0098, http://www.tankonyvtar.hu/hu/tartalom/tamop412A/2011-0098_college_geometry/index.html

John Roe: Elementary Geometry, Oxford University Press, 1993.

Vincze Csaba: Convex Geometry, University of Debrecen, 2013, TÁMOP-4.1.2.A/1-11/1-2011-0025.

**Schedule:**

*1st week*

The solution of geometric problems by vector algebra.

*2nd week*

The barycenter of a triangle and a tetrahedron.

*3rd week*

Linear dependency and independency, basis, coordinates.

*4th week*

The simple ratio.

*5$^{th}$ week*

The ellipse as the affine image of a circle. The area of an ellipse.

*6$^{th}$ week*

Compass-and-ruler constructions.

*7$^{th}$ week*

The coordinate geometry of conics.

*8$^{th}$ week*

Scalar, vector and mixed products.

*9$^{th}$ week*

Computations in higher dimensional Euclidean vector spaces. Hyperplanes and reflections (analytic description).

*10$^{th}$ week*

Reflections about lines and planes, rotations around lines and points

*11$^{th}$ week*

Lines, circles, planes and spheres. Intersections, distance and angles.

*12$^{th}$ week*

Curves and surfaces (ruled- and revolution surfaces) implicit forms and parametrizations.

*13$^{th}$ week*

Rolling without slipping: the cycloid. Twisted surfaces.

*14$^{th}$ week*

Convex geometry.

**Requirements:**

*- for a signature*

Participation at practice classes is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence.

*- for a grade*

During the semester there are two tests. The minimum requirement for a grade is to get more than 60% of the total score of the two tests, separately. The grade is given according to the following table:

| Percent | Grade |
|---|---|
| 0-60 | fail (1) |
| 61-70 | pass (2) |
| 71-80 | satisfactory (3) |
| 81-90 | good (4) |
| 91-100 | excellent (5) |

If the score of any test is below 60%, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS. The final grade is the average of the grades of the two tests by the usual rules of rounding.

*-an offered grade:* -

**Person responsible for course:** Dr. Csaba Vincze, associate professor, PhD

**Lecturer:** Dr. Csaba Vincze, associate professor, PhD

| **Title of course**: Differential geometry<br>**Code**: TTMBE0303 | **ECTS Credit points: 3** |
|---|---|
| **Type of teaching, contact hours**<br>- lecture: 2 hours/week<br>- practice: -<br>- laboratory: - | |
| **Evaluation:** exam | |
| **Workload (estimated), divided into contact hours:**<br>- lecture: 28 hours<br>- practice: -<br>- laboratory: -<br>- home assignment: 22 hours<br>- preparation for the exam: 40 hours<br>Total: 90 hours | |
| **Year, semester**: 3rd year, 1st semester | |
| **Its prerequisite(s)**: TTMBE0302, TTMBE0204 | |
| **Further courses built on it**: - | |

**Topics of course**

Introduction to the basic concepts, methods and theories of classical differential geometry: A curve in the 2- or 3-dimensional Euclidean plane space. Curvature and torsion. Surfaces in the Euclidean space. The first and second fundamental form of parameterized surfaces. Measurement on a surface. Curvature of a surface. Relationships between basic fundamental quantities. Parallelism on a surface. Geodesics. Variation of arc length. The minimizing properties of geodesics.

**Literature**

*Compulsory: -*
*Recommended:*
M. do Carmo: Differential Geometry of curves and Surfaces,
M. Spivak: A Comprehensive Introduction to Differential Geometry, Vol. 3,
B. O'Neill: Elementary Differential Geometry

**Schedule:**

*1st week*

A regular smooth curve in the Euclidean space. Parametrization and reparametrization of curves. Arc-length. Natural parametrization.

*2nd week*

Curvature of regular planar curves. Frenet base. Winding number of closed curves. Fundamental theorem of planar curves.

*3rd week*

Biregular curves in the 3-dimensional Euclidean space. Frenet-basis, curvature and torsion of a biregular curve.

*4th week*

Cartan matrix associated to a biregular curve. Fundamental theorem of curves in the 3-dimensional Euclidean space.

*5th week*

Surfaces in the Euclidean space. Parametrization of surfaces. Tangent space, normal direction at a point.

*6th week*

Measurement on surfaces. The first fundamental form of parametrized elemental surfaces. The arc length and angle of curves on surfaces. Surface area.

*7th week*

The second fundamental form of parametrized surfaces. Osculating paraboloids, Dupin indicatrix, classification of points on surfaces.

*8th week*

Gauss and Weingarten maps. Curvature curves on surfaces. Normal curvature. Meusnier's theorem.

*9th week*

Main or principal curvatures and directions. Rodriguez's theorem, Euler's formula. Product curvature and average curvature.

*10th week*

The Gauss basis associated to the parametrized surfaces. Christoffel symbols. Relationships between basic fundamental quantities. Theorema egregium. Bonnet's theorem.

*11th week*

Parallelism on a surface. Parallel translation along a curve. Geodesic curves on a surface. Geodesic curvature.

*12th week*

Variation problem of arc length. The minimizing properties of geodesics.

*13th week*

The Gauss-Bonnet theorem.

*14th week*

Surfaces with constant curvature.

**Requirements:**

Only students who have signature from the practical part can take part of the exam. The exam is written. The grade is given according to the following table:

| Score | Grade |
|---|---|
| 0-49 | fail (1) |
| 50-62 | pass (2) |
| 63-74 | satisfactory (3) |
| 75-86 | good (4) |
| 87-100 | excellent (5) |

**Person responsible for course:** Dr. Zoltán Muzsnay, associate professor, PhD

**Lecturer:** Dr. Zoltán Muzsnay, associate professor, PhD

| **Title of course**: Differential geometry<br>**Code**: TTMBG0303 | **ECTS Credit points: 2** |
|---|---|

**Type of teaching, contact hours**
- lecture: -
- practice: 2 hours/week
- laboratory: -

**Evaluation:** mid-semester grade

**Workload (estimated), divided into contact hours:**
- lecture: -
- practice: 42 hours
- laboratory: -
- home assignment: 18 hours
- preparation for the exam:

Total: 60 hours

**Year, semester**: 3rd year, 1st semester

**Its prerequisite(s)**: TTMBE0302, TTMBE0204

**Further courses built on it**: -

**Topics of course**

Introduction to the basic concepts, methods and theories of classical differential geometry: A curve in the 2- or 3-dimensional Euclidean plane space. Curvature and torsion. Surfaces in the Euclidean space. The first and second fundamental form of parameterized surfaces. Measurement on a surface. Curvature of a surface. Relationships between basic fundamental quantities. Parallelism on a surface. Geodesics. Variation of arc length. The minimizing properties of geodesics.

**Literature**

*Compulsory: -*
*Recommended:*
M. do Carmo: Differential Geometry of curves and Surfaces,
M. Spivak: A Comprehensive Introduction to Differential Geometry, Vol. 3,
B. O'Neill: Elementary Differential Geometry

**Schedule:**

*1st week*

A regular smooth curve in the Euclidean space. Parametrization and reparametrization of curves. Arc-length. Natural parametrization.

*2nd week*

Curvature of regular planar curves. Frenet base. Winding number of closed curves. Fundamental theorem of planar curves.

*3rd week*

Biregular curves in the 3-dimensional Euclidean space. Frenet-basis, curvature and torsion of a biregular curve.

*4th week*

Cartan matrix associated to a biregular curve. Fundamental theorem of curves in the 3-dimensional Euclidean space.

*5th week*

Surfaces in the Euclidean space. Parametrization of surfaces. Tangent space, normal direction at a point.

*6th week*

Measurement on surfaces. The first fundamental form of parametrized elemental surfaces. The arc length and angle of curves on surfaces. Surface area.

*7th week*

The second fundamental form of parametrized surfaces. Osculating paraboloids, Dupin indicatrix, classification of points on surfaces.

*8th week*

Test. Gauss and Weingarten maps. Curvature curves on surfaces. Normal curvature. Meusnier's theorem.

*9th week*

Main or principal curvatures and directions. Rodriguez's theorem, Euler's formula. Product curvature and average curvature.

*10th week*

The Gauss basis associated to the parametrized surfaces. Christoffel symbols. Relationships between basic fundamental quantities. Theorema egregium. Bonnet's theorem.

*11th week*

Parallelism on a surface. Parallel translation along a curve. Geodesic curves on a surface. Geodesic curvature.

*12th week*

Variation problem of arc length. The minimizing properties of geodesics.

*13th week*

Surfaces with constant curvature.

*14th week*

Test

**Requirements:**

*- for a signature*

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence.

*- for a grade*

During the semester one test is written. The grade is given according to the following table:

| Score | Grade |
|---|---|
| 0-49 | fail (1) |
| 50-59 | pass (2) |
| 60-74 | satisfactory (3) |
| 75-84 | good (4) |
| 85-100 | excellent (5) |

Students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

**Person responsible for course:** Dr. Zoltán Muzsnay, associate professor, PhD

**Lecturer:** Dr. Zoltán Muzsnay, associate professor, PhD

| **Title of course**: Vector Analysis<br>**Code**: TTMBE0304 | **ECTS Credit points: 3** |
|---|---|

**Type of teaching, contact hours**
- lecture: 2 hours/week
- practice: -
- laboratory: -

**Evaluation:** exam

**Workload (estimated), divided into contact hours:**
- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 62 hours

Total: 90 hours

**Year, semester**: 3rd year, 2nd semester

**Its prerequisite(s)**: TTMBE0204, TTMBG0304(p)

**Further courses built on it**: -

**Topics of course**

Scalar fields: level curves and surfaces. The gradient and its geometric interpretation. Vector fields, the invariants of the Jacobian matrix: divergence and rotation (the vector invariant of the skew-symmetric part of the Jacobian). The Laplace operator. Parametrized curves, line integrals and work done. Stokes' theorem and its applications in the plane: conservative vector fields and potential (path independence for line integrals, rotation-free vector fields, exact differential equations). Parametrized surfaces, surface integrals: the fluxus. Gauss-Ostrogradsky theorem and Stokes' theorem in the space. Divergence and flux density. Rotation and circulation density. Identities and computational rules for vector operators: gradient, divergence and rotation. The derivative of the determinant function: the special linear group and its Lie algebra. The orthogonal group and its Lie algebra. Displacement fields: strain and rotational tensors. Integral curves and flows. Divergence-free vector fileds (Liouville theorem, incompressible flows). Harmonic, subharmonic and superharmonic functions, the maximum principle.

**Literature**


Compulsory/Recommended Readings:
M. H. Protter, H. F. Weinberger: Maximum Principles in Differential Equations, Springer New York, 1984.
E. C. Young: Vector and Tensor Analysis, New York : M. Dekker, 1978.


**Schedule:**

*1st week*
Scalar fields: level curves and surfaces. The gradient and its geometric interpretation.

*2nd week*
Vector fields, the invariants of the Jacobian matrix: divergence and rotation (the vector invariant of the skew-symmetric part of the Jacobian). The Laplace operator.

*3rd week*
Parametrized curves, line integrals and work done.

*4th week*

Stokes' theorem and its applications in the plane: conservative vector fields and potential (path independence for line integrals, rotation-free vector fields, exact differential equations).

*5th week*

Parametrized surfaces, surface integrals: the fluxus.

*6th week*

Gauss-Ostrogradsky theorem.

*7th week*

Stokes' theorem in the space.

*8th week*

Divergence and flux density. Rotation and circulation density.

*9th week*

Identities and computational rules for vector operators: gradient, divergence and rotation.

*10th week*

The derivative of the determinant function: the special linear group and its Lie algebra.

*11th week*

The orthogonal group and its Lie algebra.

*12th week*

Displacement fields: strain and rotational tensors.

*13th week*

Integral curves and flows. Divergence-free vector fileds (Liouville theorem, incompressible flows).

*14th week*

Harmonic, subharmonic and superharmonic functions, the maximum principle.

**Requirements:**

*- for a signature*

*- for a grade*

Attendance at **lectures** is recommended, but not compulsory. The course ends in an **examination**. A practice grade at least 2 (pass) is a prerequisite to take an exam. The minimum requirement of the exam is more than 60 %. The grade is given according to the following table:

| Percent | Grade |
|---|---|
| 0-60 | fail (1) |
| 61-70 | pass (2) |
| 71-80 | satisfactory (3) |
| 81-90 | good (4) |
| 91-100 | excellent (5) |

*-an offered grade:*

**Person responsible for course:** Dr. Csaba Vincze, associate professor, PhD

**Lecturer:** Dr. Csaba Vincze, associate professor, PhD

| **Title of course**: Vector analysis<br>**Code**: TTMBG0302 | **ECTS Credit points: 2** |
|---|---|
| **Type of teaching, contact hours**<br>- lecture:<br>- practice: 2 hours/week<br>- laboratory: - | |
| **Evaluation:** mid-semester grade | |
| **Workload (estimated), divided into contact hours:**<br>- lecture: 28 hours<br>- practice: -<br>- laboratory: -<br>- home assignment: -<br>- preparation for the exam: 32 hours<br>Total: 60 hours | |
| **Year, semester**: 3rd year, 2nd semester | |
| **Its prerequisite(s)**: TTMBE0204, TTMBG0304(p) | |
| **Further courses built on it**: - | |

**Topics of course**

Scalar fields. Gradient and its geometric interpretation (level sets). Vector fields. Divergence and rotation. Laplacian. Identities. Parameterized curves. Line integrals. Stokes theorem in the plane and its applications: conservativity, potential, exact differential equations. Paremeterized surfaces. Surface integrals. Gauss-Ostrogradsky theorem and its consequences. Stokes theorem in the space and its applications. Newton's law of gravitation and its consequences: the conservativity of the gravitational field. Kepler's laws. Coordinate transforms and Jacobian (polar coordinates in the plane and in the space, cylindrical coordinates). The special linear group. The orthogonal group and its tangent space at the identity. Vector fields, integral curves and flows. Applications in the theory of differential equations. The maximum principle and its applications.

**Literature**

M. H. Protter, H. F. Weinberger: Maximum Principles in Differential Equations, Springer New York, 1984.
E. C. Young: Vector and tensor analysis, New York : M. Dekker, 1978.

**Schedule:**

*1st week*

Scalar fields and the gradient.

*2nd week*

Vector fields. Divergence and rotation. Laplacian.

*3rd week*

Identities and computational rules.

*4th week*

The parameterization of curves. Line integrals.

*5th week*

Stokes theorem in the plane and its applications: conservativity, potential, exact differential equations.

*6th week*

The parametrization of surfaces. Surface integrals.

*7th week*

Gauss-Ostrogradsky theorem and Stokes theorem in the space.

*8th week*

Newton's law of gravitation and its consequences: the conservativity of the gravitational field.

*9th week*

Kepler's laws.

*10th week*

Coordinate transforms and Jacobian (polar coordinates in the plane and in the space, cylindrical coordinates ).

*11th week*

The special linear group. The orthogonal group and its tangent space at the identity.

*12th week*

Vector fields, integral curves and flows.

*13th week*

Applications in the theory of differential equations.

*14th week*

The maximum principle. Harmonic-, subharmonic and superharmonic functions.

**Requirements:**

*- for a signature*

Participation at practice classes is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence.

*- for a grade*

During the semester there are two tests. The minimum requirement for a grade is to get more than 60% of the total score of the two tests, separately. The grade is given according to the following table:

| Percent | Grade |
|---|---|
| 0-60 | fail (1) |
| 61-70 | pass (2) |
| 71-80 | satisfactory (3) |
| 81-90 | good (4) |
| 91-100 | excellent (5) |

If the score of any test is below 60%, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS. The final grade is the average of the grades of the two tests by the usual rules of rounding.

*-an offered grade:* -

**Person responsible for course:** Dr. Csaba Vincze, associate professor, PhD

**Lecturer: :** Dr. Csaba Vincze, associate professor, PhD

| **Title of course**: Measure and integral theory<br>**Code**: TTMBE0205 | **ECTS Credit points:** 3 |
|---|---|

**Type of teaching, contact hours**

- lecture: 2 hours/week
- practice: -
- laboratory: -

**Evaluation:** exam

**Workload (estimated), divided into contact hours:**

- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 62 hours

Total: 90 hours

**Year, semester**: $2^{nd}$ year, $2^{nd}$ semester

**Its prerequisite(s)**: TTMBE0203

**Further courses built on it**: TTMBE0401, TTMBG0401

**Topics of course**

Measure spaces and measures, their properties. Outer measures, pre-measures. Construction of measures. Lebesgue measure and its topological properties. Borel sets. The structure theorem of open sets. Approximation theorem. The properties of the Cantor set. Existence of non Lebesgue measurable sets. The Lebesgue–Stieltjes measure. Measurable functions and their basic properties, Lusin's theorem. Sequences of measurable functions. heorems of Lebesgue and Egoroff, Riesz's theorem on convergence in measure, approximation lemma. The Lebesgue integral of non-negative measurable functions. Beppo Levi's theorem, Fatou's lemma. The relation between the integral and the sum. Integrable functions. Lebesgue's majorized convergence theorem. The σ-additivity and the absolute continuity of the integral. The Lebesgue integral of complex functions. Lp spaces. Minkowski and Hölder inequality. The Riesz–Fischer theorem. The relation between the Riemann and the Lebesgue integral. Fubini's theorem. The n-dimensional Lebesgue measure. Lebesgue's differentiability theorem. Functions of bounded variation and absolute continuous functions. Basic properties of antiderivatives. The Newton–Leibniz formula.

**Literature**

*Compulsory:*

- H. Federer (1969): Geometric Measure Theory, Springer-Verlag
- Paul R. Halmos (1950): Measure Theory, D. Van Nostrand Company, Inc.

*Recommended:*

- Anthony W. Knapp (2005): Basic Real Analysis, Birkhauser

**Schedule:**

*$1^{st}$ week*

The definition of measure spaces and measures, and their most important properties.

*$2^{nd}$ week*

Outer measures and their characterization, the notion of premeasures. Construction of measures. The definition of the Lebesgue measure.

*3rd week*

The notion of the Lebesgue measure and its most important topological properties. Borel sets. The structure theorem of open sets. Approximation theorem.

*4th week*

The construction and most important properties of the Cantor set. Existence of not Lebesgue measurable sets.

*5th week*

Lebesgue-Stieltjes measure. The definition and fundamental properties of measurable functions, Luzin theorem.

*6th week*

Sequences of measurable functions. The definition of convergence in measure and results related to it: theorems of Lebesgue and Egoroff, the selection theorem of Riesz, approximation lemma.

*7th week*

The Lebesgue integral of nonnegative measurable functions and its basic properties. The theorem of Beppo Levi. Fatou lemma.

*8th week*

The relation between the integral and the sum. Integrable functions and their fundamental properties.

*9th week*

Lebesgue's majorized convergence theorem. The σ-additivity and the absolute continuity of the integral.

*10th week*

The Lebesgue integral of complex functions. Lp spaces. Minkowski and Hölder inequality. The Riesz–Fischer theorem.

*11th week*

The relation between the Riemann and the Lebesgue integral. Fubini's theorem. The n-dimensional Lebesgue measure.

*12th week*

Lebesgue's differentiability theorem.

*13th week*

Functions of bounded variation and absolute continuous functions. Basic properties of antiderivatives.

| |
|---|
| *14$^{th}$ week*<br>The Newton-Leibniz formula. |
| **Requirements:**<br>The course ends in an **oral examination**. The process of the exam is as follows. First, a topic out of cca. eight is chosen randomly. The list of possible topics is made available for the students before the exam period. Then, the chosen topic should be elaborated in writing. Based partly on what has been written, an oral discussion of the topic follows, which also contains a few questions about other topics. The performance of the student during the exam is evaluated by a grade.<br><br>Attendance of lectures is recommended, but not obligatory. |
| **Person responsible for course:** Dr. Gergő Nagy, assistant professor, PhD |
| **Lecturer:** Dr. Gergő Nagy, assistant professor, PhD |

| **Title of course**: Probability theory<br>**Code**: TTMBE0401 | **ECTS Credit points: 4** |
|---|---|

**Type of teaching, contact hours**
- lecture: 3 hours/week
- practice: -
- laboratory: -

**Evaluation:** exam

**Workload (estimated), divided into contact hours:**
- lecture: 42 hours
- practice: -
- laboratory: -
- home assignment: 40 hours
- preparation for the exam: 38 hours

Total: 120 hours

**Year, semester**: 2nd year, 1st semester

**Its prerequisite(s)**: TTMBE0205

**Further courses built on it**: -

**Topics of course**

Probability, random variables, distributions. Asymptotic theorems of probability theory.

**Literature**

*Compulsory:*
- A. N. Shiryayev: Probability, Springer-Verlag, Berlin, 1984.
- Ash, R. B.: Real Analysis and Probability. Academic Press, New York-London, 1972.
- Bauer, H.: Probability Theory. Walter de Gruyter, Berlin-New York. 1996.

**Schedule:**

*1st week*

Statistical observations. Numerical and graphical characteristics of the sample. Relative frequency, events, probability. Classical probability. Finite probability space.

*2nd week*

Kolmogorov's probability space. Properties of probability. Finite and countable probability spaces. Conditional probability, independence of events. Borel-Cantelli lemma.

*3rd week*

Total probability theorem, the Bayes rule. Discrete random variables. Expectation, Standard deviation. Binomial, hypergeometric, and Poisson distributions.

*4th week*

Random variables, distribution, cumulative distribution function. Absolutely continuous distribution, probability density function. The general notion of distribution.

*5th week*

Expectation, variance and median. Uniform, exponential, normal distributions.

*6th week*

Joint distribution function and joint probability density function of random variables. Independent random variables, correlation coefficient.

| |
|---|
| *7th week*<br>Multivariate distributions. Expectation vector and variance matrix of random a random vector. Independence of random variables.<br>*8th week*<br>Multivariate normal distribution, concentration ellipsoid. Sample from normal distribution. Chi-squared, Student's t, F-distributions.<br>*9th week*<br>Weak law of large numbers. Almost sure convergence, convergence in distribution, convergence in probability, Lp convergence.<br>*10th week*<br>Kolmogorov's inequality. Kolmogorov's three-series theorem. Strong laws of large numbers.<br>*11th week*<br>Characteristic function and its properties. Inversion formulas. Continuity theorem<br>*12th week*<br>Central limit theorem. Law of the iterated logarithm. Arcsine laws.<br>*13th week*<br>Conditional distribution function, conditional density function, conditional expectation.<br>*14th week*<br>Comparison of the limit theorems. |
| **Requirements:**<br>*- for a grade* |
| **Person responsible for course:** Prof. Dr. István Fazekas, university professor, DSc |
| **Lecturer:** Prof. Dr. István Fazekas, university professor, DSc |

| **Title of course**: Probability theory<br>**Code**: TTMBG0401 | **ECTS Credit points: 2** |
|---|---|

**Type of teaching, contact hours**
- lecture: -
- practice: 2 hours/week
- laboratory: -

**Evaluation:** mid-semester grade

**Workload (estimated), divided into contact hours:**
- lecture: -
- practice: 28 hours
- laboratory: -
- home assignment: 16 hours
- preparation for the exam: 16 hours

Total: 60 hours

**Year, semester**: 2nd year, 1st semester

**Its prerequisite(s)**: TTMBE0205

**Further courses built on it**: -

**Topics of course**

Probability, random variables, distributions. Asymptotic theorems of probability theory.

**Literature**

Compulsory:
- A. N. Shiryayev: Probability, Springer-Verlag, Berlin, 1984.
- Ash, R. B.: Real Analysis and Probability. Academic Press, New York-London, 1972.
- Bauer, H.: Probability Theory. Walter de Gruyter, Berlin-New York. 1996.

**Schedule:**

*1st week*

Statistical observations. Numerical and graphical characteristics of the sample. Relative frequency, events, probability. Classical probability. Finite probability space.

*2nd week*

Kolmogorov's probability space. Properties of probability. Finite and countable probability spaces. Conditional probability, independence of events. Borel-Cantelli lemma.

*3rd week*

Total probability theorem, the Bayes rule. Discrete random variables. Expectation, Standard deviation. Binomial, hypergeometric, and Poisson distributions.

*4th week*

Random variables, distribution, cumulative distribution function. Absolutely continuous distribution, probability density function. The general notion of distribution.

*5th week*

Expectation, variance and median. Uniform, exponential, normal distributions.

*6th week*

Joint distribution function and joint probability density function of random variables. Independent random variables, correlation coefficient.

| |
|---|
| *7th week*<br>Multivariate distributions. Expectation vector and variance matrix of random a random vector. Independence of random variables.<br>*8th week*<br>Multivariate normal distribution, concentration ellipsoid. Sample from normal distribution. Chi-squared, Student's t, F-distributions.<br>*9th week*<br>Weak law of large numbers. Almost sure convergence, convergence in distribution, convergence in probability, Lp convergence.<br>*10th week*<br>Kolmogorov’s inequality. Kolmogorov's three-series theorem. Strong laws of large numbers.<br>*11th week*<br>Characteristic function and its properties. Inversion formulas. Continuity theorem<br>*12th week*<br>Central limit theorem. Law of the iterated logarithm. Arcsine laws.<br>*13th week*<br>Conditional distribution function, conditional density function, conditional expectation.<br>*14th week*<br>Comparison of the limit theorems. |
| **Requirements:**<br>*- for a grade* |
| **Person responsible for course:** Prof. Dr. István Fazekas, university professor, DSc |
| **Lecturer:** Prof. Dr. István Fazekas, university professor, DSc |

| **Title of course**: Introduction to informatics<br>**Code**: TTMBG0601 | **ECTS Credit points:** 2 |
|---|---|

**Type of teaching, contact hours**

- lecture: -
- practice: 3 hours/week
- laboratory: -

**Evaluation:** mid-semester grade

**Workload (estimated), divided into contact hours:**

- lecture: -
- practice: 42 hours
- laboratory: -
- home assignment: -
- preparation for the exam: 18 hours

Total: 60 hours

**Year, semester**: 1st year, 2nd semester

**Its prerequisite(s)**: -

**Further courses built on it**: -

**Topics of course**

An introduction to LaTeX, a document preparation system for high-quality typesetting. Typesetting of complex mathematical formulas in LaTeX. Presentation creation using the Beamer class. Writing a formal or business letter in LaTeX. Using the moderncv class for typesetting curricula vitae. The memoire class, a tool to create BSc/MSc thesis. Introduction to SageMath, a computer algebra package. The Jupyter Notebook interface and the SageMathCloud. Basic tools, assignment, equality, and arithmetic. Boolean expressions, loops, lists and sets. Writing functions in SageMath.

**Literature**

*Compulsory:*

-

*Recommended:*

T. Oetiker: The Not So Short Introduction to LaTeX.

Gregory Bard: SageMath for Undergraduates (http://www.gregorybard.com/Sage.html)

**Schedule:**

*1st week*

Basic usage of LaTeX. MikTeX and TeXLive distributions. The TeXmaker editor.

*2nd week*

Preparing LaTeX documents, basic mathematical formulas in LaTeX.

*3rd week*

Complex mathematical formulas, matrices, tables in LaTeX.

*4th week*

Presentation in LaTeX, the beamer package and its usage. Special LaTeX commands in presentations.

*5th week*

Definitions, theorems in LaTeX, the memoire package and its usage to prepare thesis. The bibtex package.

*6th week*

The moderncv package, curriculum vitae and formal letter in LaTeX.

*7th week*

First test.

*8th week*

The SageMath computer algebra package, basic mathematical usage.

*9th week*

Functions related to the rings of integers, computing the gcd and the extended euclidean algorithm.

*10th week*

Polynomial rings in SageMath, rational functions and related commands.

*11th week*

Sets and lists in SageMath, basic operations, loops in lists and sets.

*12th week*

Trigonometric functions in SageMath, expanding and simplifying expressions.

*13th week*

Defining functions in SageMath, preparing plots. Solving special equations, systems of equations.

*14th week*

Second test.

**Requirements:**

*- for a signature*

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

*- for a grade*

The course is evaluated on the basis of two written tests during the semester. The grade is given according to the following table:

| Total Score (%) | Grade |
|---|---|
| 0 – 50 | fail (1) |
| 51 – 60 | pass (2) |
| 61 – 70 | satisfactory (3) |
| 71 – 85 | good (4) |
| 86 – 100 | excellent (5) |

If a student fail to pass at first attempt, then a retake of the tests is possible.

*-an offered grade:*

It is not possible to obtain an offered grade in this course.

**Person responsible for course:** Dr. Szabolcs Tengely, associate professor, PhD

**Lecturer:** Dr. Szabolcs Tengely, associate professor, PhD

| **Title of course**: Programming languages<br>**Code**: TTMBG0602 | **ECTS Credit points:** 2 |
|---|---|

**Type of teaching, contact hours**

- lecture: -
- practice: 2 hours/week
- laboratory: -

**Evaluation:** practice

**Workload (estimated), divided into contact hours:**

- lecture: -
- practice: 28 hours
- laboratory: -
- home assignment: 16 hours
- preparation for the exam: 16 hours

Total: 60 hours

**Year, semester**: 1st year, 2nd semester

**Its prerequisite(s)**: -

**Further courses built on it**: -

**Topics of course**

Brief introduction to programming, programming languages, and architecture in general. Sequential, conditional, and repeated execution, reuse. Values and types, expressions. Container data types and standard uses. Reading and writing files. Text processing with string methods and regular expressions. Object-oriented design in practice. Basics of networked programming, working with data over the internet. Fundamentals of using databases and visualisation of data. Complex programming exercises.

**Literature**

*Compulsory:*

-

*Recommended:*

Charles Severance, Python for Everybody: Exploring Data in Python 3, 2016.

Allen B. Downey, Think Python, 2012

**Schedule:**

*1st week*

General introduction to programming and programming languages. Simplified structure of programs: sequential, conditional, and repeated execution, reuse. Flowcharts, errors and debugging. About Python.

*2nd week*

Values and types, standard types in Python, differences between classes and types. Variables: assignment, naming conventions, and aliasing. Expressions: numerical and Boolean operations, orders of operations, short-circuit evaluation of logical expressions.

*3rd week*

Blocks and indentation in Python. Conditional execution: single conditionals, alternative executions, chained and nested conditionals, try and except. Repeated execution: definite loops, the range type, indefinite loops, infinite loops, and loop controls.

*4th week*

Functions: function calls, arguments and parameters, built-in and user-defined functions, fruitful and void functions, modules.

*5th week*

Classification of container data types: iterable, mutable, and ordered types. Strings, lists, tuples, sets, and dictionaries and their basic roles.

*6th week*

Files: open and close, read and write, creating new files and directories. Parsing strings with string methods.

*7th week*

Regular expressions as a formal language and as strings with standard and meta characters. Parsing strings with regular expression methods.

*8th week*

Object-oriented design: goals, principles, and patterns. Instances and methods: accessor, mutator, and manager methods. Classes in Python.

*9th week*

Networked programming: a brief introduction to HTML, XML, and JSON. Retrieving and processing content over the internet.

*10th week*

Networked programming continued.

*11th week*

Databases: a brief introduction to databases and SQL. Reading, writing, and processing data from databases.

*12th week*

Visualization of data.

*13th week*

Extensive programming class.

*14th week*

Final exam.

**Requirements:**

*- for a signature*

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

*- for a grade*

The course is evaluated on the basis of one written test and a homework project submitted during the semester. The grade is given according to the following table:

| **Total Score (%)** | **Grade** |
|---|---|
| 0 – 50 | fail (1) |
| 51 – 60 | pass (2) |
| 61 – 70 | satisfactory (3) |
| 71 – 80 | good (4) |
| 81 – 100 | excellent (5) |

If a student fail to pass at first attempt, then a retake of the test is possible.

*-an offered grade:*

It is not possible to obtain an offered grade in this course.

| |
|---|
| **Person responsible for course:** Dr. András Bazsó, assistant professor, PhD |
| **Lecturer:** Dr. András Bazsó, assistant professor, PhD |

| **Title of course**: Algorithms<br>**Code**: TTMBE0606 | **ECTS Credit points:** 3 |
|---|---|

**Type of teaching, contact hours**
- lecture: 2 hours/week
- practice: -
- laboratory: -

**Evaluation:** exam

**Workload (estimated), divided into contact hours:**
- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 62 hours

Total: 90 hours

**Year, semester**: 1st year, 2nd semester

**Its prerequisite(s)**: TTMBE0107

**Further courses built on it**: -

**Topics of course**

Classification of programming languages. Multi-character symbols. Data types. Instruction types. Loops. Subprograms. The role of algorithms in computing. Functions, recursive functions. Probabilistic analysis. Randomized algorithms. Heap, heapsort. Quicksort. Sorting in linear time. Elementary data structures.

**Literature**

*Compulsory:*
-
*Recommended:*
T. H. Cormen, C. E. Leiserson, R. L. Rivest, C. Stein: Introduction to Algorithms, MIT Press, Cambridge, 2009 (3rd ed.)
István Juhász: Programming Languages.,
http://www.tankonyvtar.hu/en/tartalom/tamop425/0046_programming_languages/index.html

**Schedule:**

*1st week*

Introduction, foundations. Classification of programming languages.

*2nd week*

Multi-character symbols, symbolic names, labels, comments, literals, (constants.) Data types (simple, composite and pointer types).

*3rd week*

Assignment statements, the empty statements, the GOTO statement, selection statements, conditional statements, case/switch statement.

*4th week*

Loop statements, conditional loops, count-controlled loops, enumeration-controlled loops, infinite loops, composite loops.

*5th week*

Subprograms, the call chain and recursion, secondary entry points, parameter evaluation and parameter passing, block, scope, compilation unit.

*6th week*

The role of algorithms in computing. Algorithms as a technology, insertion sort, analyzing algorithms, designing algorithms.

*7th week*

Functions, recursive functions. Grow of functions, asymptotic notation, standard notations and common functions. Recurrences, the substitution method and the recursion-tree method for solving recurrences.

*8th week*

The master method and proof of the master method.

*9th week*

Probabilistic analysis, the hiring problem, indicator random variables.

*10th week*

Randomized algorithms and further examples of probabilistic analysis.

*11th week*

Heap, heapsort, maintaining the heap property, building the heap, the heapsort algorithm, priority queues.

*12th week*

Quicksort, description of quicksort, performance of quicksort, a randomized version of quicksort, analysis of quicksort.

*13th week*

Sorting in linear time, lower bounds for sorting, counting sort, radix sort, bucket sort.

*14th week*

Elementary data structures, stacks and queues, linked lists, implementing pointers and objects, representing rooted trees.

**Requirements:**

- *for a signature*

If the student fail the course TTMBG0606, then the signature is automatically denied.

- *for a grade*

The course ends in oral examination. The grade is given according to the following table:

| Total Score (%) | Grade |
|---|---|
| 0 – 50 | fail (1) |
| 51 – 60 | pass (2) |
| 61 – 70 | satisfactory (3) |
| 71 – 85 | good (4) |
| 86 – 100 | excellent (5) |

*-an offered grade:*

It is possible to obtain an offered grade on the basis of two written test during the semester. The grade is given according to the following table:

| Total Score (%) | Grade |
|---|---|
| 0 – 50 | fail (1) |
| 51 – 60 | pass (2) |
| 61 – 70 | satisfactory (3) |
| 71 – 85 | good (4) |

| | |
|---|---|
| 86 – 100 | excellent (5) |

**Person responsible for course:** Dr. Nóra Györkös-Varga, assistant professor, PhD

**Lecturer:** Dr. Nóra Györkös-Varga, assistant professor, PhD

| **Title of course**: Algorithms<br>**Code**: TTMBG0606 | **ECTS Credit points:** 2 |
|---|---|

**Type of teaching, contact hours**

- lecture: -
- practice: 2 hours/week
- laboratory: -

**Evaluation:** practice

**Workload (estimated), divided into contact hours:**

- lecture: -
- practice: 28 hours
- laboratory: -
- home assignment: -
- preparation for the exam: 32 hours

Total: 60 hours

**Year, semester**: 1st year, 2nd semester

**Its prerequisite(s)**: TTMBE0107

**Further courses built on it**: -

**Topics of course**

Classification of programming languages. Multi-character symbols. Data types. Instruction types. Cycles. Subprograms. The role of algorithms in computing. Functions, recursive functions. Probabilistic analysis. Randomized algorithms. Heap, heapsort. Quicksort. Sorting in linear time. Elementary data structures.

**Literature**

*Compulsory:*

-

*Recommended:*

T. H. Cormen, C. E. Leiserson, R. L. Rivest, C. Stein: Introduction to Algorithms, MIT Press, Cambridge, 2009 (3rd ed.)

István Juhász: Programming Languages., http://www.tankonyvtar.hu/en/tartalom/tamop425/0046_programming_languages/index.html

**Schedule:**

*1st week*

Presentation of procedural and object-oriented languages, emphasizing the main differences and presentation of structures, parts of methods.

*2nd week*

Presentation of data types (simple, composite and special), emphasizing the main differences of the types static and dynamic. Using the simpler and known data types (array, chain, list, structure), their creation from simple types.

*3rd week*

Description of main types of statements, the difference of selection statements. Representing conditional statements (if-else) and case/switch statement (if-else if, or switch); presentation of the differences of „if-else if” and „switch” in case/switch statement. Recapitulate and exercise of notations and logical foundations required for conditional statements.

*4th week*

Loop statements, conditional loops, count-controlled loops, enumeration-controlled loops, infinite loops, composite loops. Programming loops „while” and „do-while”, investigation of effect of different initialization and termination conditions concerning certain problems.

*5th week*

Functions, planning of methods, determination of return values. Connection, linking of functions and methods. Presentation of recursive functions through some examples (e.g. Fibonacci sequence). Simultaneous determination of different return values with indicators.

*6th week*

The role of algorithms in computing. Algorithms as a technology, insertion sort, bubble sort, analyzing algorithms, designing algorithms. Presentation and examination of different types of the (above) sorts with reference to efficiency.

*7th week*

Grow of functions, asymptotic notation, standard notations and common functions. Recurrences, the substitution method and the recursion-tree method for solving recurrences.

*8th week*

The master method, practical importance of the master method.

*9th week*

Probabilistic analysis, the hiring problem, indicator random variables.

*10th week*

Randomized algorithms and further examples of probabilistic analysis.

*11th week*

Heap, heapsort, maintaining the heap property, building the heap, the heapsort algorithm, priority queues.

*12th week*

Quicksort, description of quicksort, performance of quicksort, a randomized version of quicksort, analysis of quicksort.

*13th week*

Sorting in linear time, lower bounds for sorting; programming the counting sort, radix sort, bucket sort. Presentation of foundation of Hash functions and using them for sort of certain array.

*14th week*

Elementary data structures, stacks and queues, linked lists, implementing pointers and objects, representing rooted trees.

**Requirements:**

*- for a signature*

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

*- for a grade*

The course is evaluated on the basis of two written tests during the semester. The grade is given according to the following table:

| Total Score (%) | Grade |
|---|---|
| 0 – 50 | fail (1) |
| 51 – 60 | pass (2) |
| 61 – 70 | satisfactory (3) |
| 71 – 85 | good (4) |
| 86 – 100 | excellent (5) |

| |
|---|
| If a student fail to pass at first attempt, then a retake of the test is possible.<br>*-an offered grade:*<br>It is not possible to obtain an offered grade in this course. |
| **Person responsible for course:** Dr. Nóra Györkös-Varga, assistant professor, PhD |
| **Lecturer:** Dr. Nóra Györkös-Varga, assistant professor, PhD |

| **Title of course**: Applied number theory<br>**Code**: TTMBE0109 | **ECTS Credit points:** 3 |
|---|---|

**Type of teaching, contact hours**

- lecture: 2 hours/week
- practice: -
- laboratory: -

**Evaluation:** exam

**Workload (estimated), divided into contact hours:**

- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 62 hours

Total: 90 hours

**Year, semester**: 2nd year, 2nd semester

**Its prerequisite(s)**: TTMBE0106

**Further courses built on it**: TTMBE0111

**Topics of course**

Basic notions of complexity theory. Some basic algorithms and their complexity. Approximation of real numbers by rationals, the theorem of Dirichlet. Liouville's theorem, a construction of transcendental numbers. Continued fractions and their properties. Finite and infinite continued fractions. Approximation with continued fractions. The LLL-algorithm and some of its applications. Pseudoprimes and their properties. Carmichael-numbers and their role in probabilistic prime tests. Euler-pseudoprimes and their properties. The Soloway-Strassen probabilistic prime test. Strong pseudoprimes and their properties. The Miller-Rabin probabilistic prime test. Deterministic prime tests, Wilson's theorem, the description of the Agrawal-Kayal-Saxena test. The birthday paradox and Pollard's ρ–method. Fermat-factorization. Factorization with a factor basis. Factorization with continued fractions.

**Literature**

*Compulsory:*

-

*Recommended:*

Neal Koblitz: A Course in Number Theory and Cryptography, Springer Verlag, 1994.

I. Niven, H. S. Zuckerman, H. L. Montgomery: An Introduction to the Theory of Numbers, Wiley, 1991.

Nigel Smart: The Algorithmic Resolution of Diophantine Equations, London Mathematical Society Student Text 41, Cambridge University Press, 1998.

**Schedule:**

*1st week*

Basic concepts of complexity theory. Some fundamental algorithms and their complexity. Solution of related problems.

*2nd week*

Approximation of real numbers by rationals, Dirichlet's theorem. Solution of related problems.

*3rd week*

Liouville's theorem, construction of transcendental numbers. Solution of related problems.

*4th week*

Continued fractions and their properties. Finite and infinite continued fractions. Solution of related problems.

*5th week*

Approximation by continued fractions. Solution of related problems.

*6th week*

The LLL-algorithm and some of its applications. Solution of related problems.

*7th week*

Pseudoprimes and their properties. Carmichael-numbers and their role in probabilistic prime tests. Solution of related problems.

*8th week*

Euler-pseudoprimes and their properties. The Soloway-Strassen probabilistic prime test. Solution of related problems.

*9th week*

Strong pseudoprimes and their properties. The Miller-Rabin probabilistic prime test. Solution of related problems.

*10th week*

Deterministic prime tests, Wilson's theorem, the Agrawal-Kayal-Saxena test. Solution of related problems.

*11th week*

The birthday paradox and Pollard's- ρ–method. Solution of related problems.

*12th week*

Fermat-factorization. The background of the method and its variants. Solution of related problems.

*13th week*

Factorization with a factor basis.. Solution of related problems.

*14th week*

Continued fraction factorization. The background of the method and its applications. Solution of related problems.

**Requirements:**

*- for a signature*

Signature is not a basis of evaluation in this course.

*- for a grade*

The course ends in oral examination. The grade is given according to the following table:

| Total Score (%) | Grade |
|---|---|
| 0 – 50 | fail (1) |
| 51 – 60 | pass (2) |
| 61 – 70 | satisfactory (3) |
| 71 – 80 | good (4) |
| 81 – 100 | excellent (5) |

*-an offered grade:*

It is not possible to obtain an offered grade in this course.

**Person responsible for course:** Prof. Dr. Lajos Hajdu, university professor, DSc

**Lecturer:** Prof. Dr. Lajos Hajdu, university professor, DSc

| **Title of course**: Algorithms in algebra and number theory<br>**Code**: TTMBG0110 | **ECTS Credit points:** 3 |
|---|---|

**Type of teaching, contact hours**

- lecture: -
- practice: 3 hours/week
- laboratory: -

**Evaluation:** practice

**Workload (estimated), divided into contact hours:**

- lecture: -
- practice: 42 hours
- laboratory: -
- home assignment: -
- preparation for the exam: 48 hours

Total: 90 hours

**Year, semester**: 2nd year, 2nd semester

**Its prerequisite(s)**: TTMBE0106

**Further courses built on it**: -

**Topics of course**

Linear algebra and applications using SageMath. Factoring polynomials over finite fields, the Berlekamp algorithm. Shamir's secret sharing algorithm. Lattices, the LLL-algorithm and applications. Number theoretic functions in SageMath. Linear Diophantine equations, the Frobenious problem. Conics and elliptic curves in SageMath.

**Literature**

*Compulsory:*

-

*Recommended:*

Victor Shoup: A Computational Introduction to Number Theory and Algebra, Cambridge University Press, 2005.

William Stein: Elementary Number Theory: Primes, Congruences, and Secrets, Springer-Verlag, 2008

**Schedule:**

*1st week*

A short introduction of SageMath (basic structures, lists, sets, programming tools).

*2nd week*

Linear algebra over the reals, complex numbers and finite fields. The Berlekamp algorithm. Computing formulas for recurrence sequences.

*3rd week*

Polynomials and matrices over finite fields, the Samir secret sharing procedure.

*4th week*

The NTRU cryptosystem and its implementation in SageMath.

*5th week*

Polinomial equations and applications.

*6th week*

Number theoretical functions in SageMath, linear Diophantine equations. Combinatorial

*7th week*

First test.

*8th week*

Lattices and the LLL-algorithm in SageMath. The knapsack problem.

*9th week*

The Frobenius problem. Solutions of the Frobenius problem via Wilf-method and Brauer-method.

*10th week*

Computations related to quadratic residues, the Legendre symbol.

*11th week*

The ternary Diophantine equation $ax^2+by^2+cz^2=0$. Descent algorithm to determine integral solutions, parametrization of rational and integral points.

*12th week*

Elliptic curves, points on elliptic curves over the rationals, finite fields. Applications of elliptic curves.

*13th week*

Points on elliptic curves over finite fields, determining the order, the baby step-giant step algorithm.

*14th week*

Second test.

**Requirements:**

*- for a signature*

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

*- for a grade*

The course is evaluated on the basis of two written tests during the semester. The grade is given according to the following table:

| Total Score (%) | Grade |
|---|---|
| 0 – 50 | fail (1) |
| 51 – 60 | pass (2) |
| 61 – 70 | satisfactory (3) |
| 71 – 85 | good (4) |
| 91 – 100 | excellent (5) |

If a student fail to pass at first attempt, then a retake of the tests is possible.

*-an offered grade:*

It is not possible to obtain an offered grade in this course.

**Person responsible for course:** Dr. Szabolcs Tengely, associate professor, PhD

**Lecturer:** Dr. Szabolcs Tengely, associate professor, PhD

| **Title of course**: Introduction to cryptography<br>**Code**: TTMBE0111 | **ECTS Credit points:** 3 |
|---|---|

**Type of teaching, contact hours**

- lecture: 2 hours/week
- practice: -
- laboratory: -

**Evaluation:** exam

**Workload (estimated), divided into contact hours:**

- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 62 hours

Total: 90 hours

**Year, semester**: 3rd year, 1st semester

**Its prerequisite(s)**: TTMBE0109

**Further courses built on it**: -

**Topics of course**

Basic cryptographic concepts. Symmetric and asymmetric cryptosystems. The Cesar and the linear cryptosystems, DES, AES. The RSA cryptosystem and the analysis of its security. The discrete logarithm problem. Algorithms for solving the discrete logarithm problem. Cryptosystems based on the discrete logarithm problem. Elliptic curve cryptography. Basic cryptographic protocols. Digital signature. The basics of PGP.

**Literature**

*Compulsory:*

-

*Recommended:*

J. Buchmann: Einführung in die Kryptographie, Springer, 1999.

N. Koblitz: A Course in Number Theory and Cryptography, Springer, 1987.

**Schedule:**

*1st week*

The models of information transfer. The Shannon model, modulator, demodulator and their parts. The two branches of cryptology: cryptography and cryptanalysis. Major applications of cryptography. Requirements of a modern cryptosystem. The notion of cryptosystem and its mathematical model.

*2nd week*

Classical cryptosystems. Symmetric versus asymmetric cryptosystems. The Caesar cryptosystem, substitution ciphers, the affine cryptosystem. Frequency analysis. Breaking the classical cryptosystems.

*3rd week*

Block-cyphers. Feistel-type ciphers. The history of the DES, requirements for a cryptosystem in those times. Description of the DES. Security of the DES. Double DES and triple DES.

*4th week*

The field GF(2^8). Operations in GF(2^8). Bytes as elements of GF(2^8). The structure of the polynomial ring GF(2^8)[x] and of the factor ring GF(2^8)[x]/(x^4+1), operations in the factor ring GF(2^8)[x]/(x^4+1).

*5th week*

The call for proposals for AES. Requirements concerning AES. The winner of the call: the Rijndael. Description of the Rijndael cryptosystem: number of rounds, round-transformation final round, round-key generation.

*6th week*

The basic idea behind the public-key cryptosystems, the infrastructure of public key cryptosystems. The idea behind the RSA cryptosystem. Description of the RSA cipher.

*7th week*

First test.

*8th week*

The security of the RSA – correct choice of the parameters. Known protocol errors and possibilities of attacking the RSA in case of wrong parametrization or programming.

*9th week*

The discrete logarithm problem. The Pohlig-Hellman algorithm, the Baby-step Giant-step algorithm, the Pollard-rho algorithm, and the Index-calculus algorithm.

*10th week*

Public-key cryptosystems based on the hardness of the discrete logarithm problem: the El Gamal cryptosystem, the Diffie-Hellmann key-exchange protocol, the Massey-Omura cryptosystem.

*11th week*

Definition of elliptic curves. Points on elliptic curves over a given field. Definition of the group of an elliptic curve. The real case. Elliptic curves over finite fields. Hasse's theorem.

*12th week*

Encoding the plaintext as a point of an elliptic curve. Cryptosystems based on the discrete logarithm problem over elliptic curves: the El Gamal cryptosystem, the Massey-Omura cryptosystem.

*13th week*

Protocols for key-exchange, digital signature and authentication. Zero-knowledge proofs.

*14th week*

Second test.

**Requirements:**

*- for a signature*

If the student fail the course TTMBG0111, then the signature is automatically denied.

*- for a grade*

The course ends in oral examination. The grade is given according to the following table:

| Total Score (%) | Grade |
|---|---|
| 0 – 50 | fail (1) |
| 51 – 60 | pass (2) |
| 61 – 70 | satisfactory (3) |
| 71 – 85 | good (4) |
| 86 – 100 | excellent (5) |

*-an offered grade:*

It is possible to obtain an offered grade on the basis of two written test during the semester. The grade is given according to the following table:

| Total Score (%) | Grade |
| --- | --- |
| 0 – 50 | fail (1) |
| 51 – 60 | pass (2) |
| 61 – 70 | satisfactory (3) |
| 71 – 85 | good (4) |
| 86 – 100 | excellent (5) |

**Person responsible for course:** Prof. Dr. Attila Bérczes, university professor, DSc

**Lecturer:** Prof. Dr. Attila Bérczes, university professor, DSc

| **Title of course**: Introduction to cryptography<br>**Code**: TTMBG0111 | **ECTS Credit points:** 2 |
|---|---|
| **Type of teaching, contact hours**<br>- lecture: -<br>- practice: 2 hours/week<br>- laboratory: - | |
| **Evaluation:** mid-semester grade | |
| **Workload (estimated), divided into contact hours:**<br>- lecture: -<br>- practice: 28 hours<br>- laboratory: -<br>- home assignment: 16 hours<br>- preparation for the exam: 16 hours<br>Total: 60 hours | |
| **Year, semester**: 3rd year, 1st semester | |
| **Its prerequisite(s)**: TTMBE0109 | |
| **Further courses built on it**: - | |

**Topics of course**

Basic cryptographic concepts. Symmetric and asymmetric cryptosystems. The Cesar and the linear cryptosystems, DES, AES. The RSA cryptosystem and the analysis of its security. The discrete logarithm problem. Algorithms for solving the discrete logarithm problem. Cryptosystems based on the discrete logarithm problem. Elliptic curve cryptography. Basic cryptographic protocols. Digital signature. The basics of PGP.

**Literature**

*Compulsory:*

-

*Recommended:*

J. Buchmann: Einführung in die Kryptographie, Springer, 1999.

N. Koblitz: A Course in Number Theory and Cryptography, Springer, 1987.

**Schedule:**

*1st week*

Introduction to the Magma computer algebra system.

*2nd week*

Realization of the Caesar cryptosystem, a substitution cypher, or an affine cryptosystem.

*3rd week*

Programming the DES in frame of group work.

*4th week*

Continuing the implementation of DES. Combining the individually produced programme-parts.

*5th week*

Computer aided computations in the field GF(2^8), the polynomial ring GF(2^8)[x] and the factor ring GF(2^8)[x]/(x^4+1) using Magma.

*6th week*

Group work: programming the encryption/decryption function of the Rijndael cryptosystem.

*7th week*

Continuing the implementation of the Rijndael cryptosystem. Combining the individually produced programme-parts.

*8th week*

Implementing the RSA cryptosystem.

*9th week*

Programming one of the algorithms for solving the DLP.

*10th week*

Implementing one of the cryptosystems based on the hardness of the DLP.

*11th week*

Defining and manipulating elliptic curves in Magma.

*12th week*

Writing a programme to encode plaintext as a point on an elliptic curve.

*13th week*

Implementing the Diffie-Hellmann key-exchange protocol.

*14th week*

Evaluation, decision of the marks of the students.

**Requirements:**

*- for a signature*

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

*- for a grade*

The course is evaluated on the basis of one written test and a homework project submitted during the semester. The grade is given according to the following table:

| Total Score (%) | Grade |
|---|---|
| 0 – 50 | fail (1) |
| 51 – 60 | pass (2) |
| 61 – 70 | satisfactory (3) |
| 71 – 85 | good (4) |
| 86 – 100 | excellent (5) |

If a student fail to pass at first attempt, then a retake of the test is possible.

*-an offered grade:*

It is not possible to obtain an offered grade in this course.

**Person responsible for course:** Prof. Dr. Attila Bérczes, university professor, DSc

**Lecturer:** Prof. Dr. Attila Bérczes, university professor, DSc

| **Title of course**: Numerical analysis<br>**Code**: TTMBE0209 | **ECTS Credit points:** 4 |
|---|---|
| **Type of teaching, contact hours**<br>- lecture: 3 hours/week<br>- practice: -<br>- laboratory: - | |
| **Evaluation:** exam | |
| **Workload (estimated), divided into contact hours:**<br>- lecture: 42 hours<br>- practice: -<br>- laboratory: -<br>- home assignment: -<br>- preparation for the exam: 78 hours<br>Total: 120 hours | |
| **Year, semester**: 2nd year, 2nd semester | |
| **Its prerequisite(s)**: TTMBE0102, TTMBE0203, TTMBG0209(p) | |
| **Further courses built on it**: - | |

**Topics of course**

Features of computions by computer, error propagation. Some important matrix transformations for solving linear systems and eigenvalue problems. Gaussian elimination and its variants: its algorithms, operational complexity, pivoting. Decompositions of matrices: Schur complement, LU decomposition, LDU decomposition, Cholesky decomposition, QR decomposition. Iterative methods for solving linear and nonlinear systems: Gauss-Seidel iteration, gradient method, conjugate gradient method, Newton method, local and global convergence, quasi-Newton method, Levenberg–Marquardt algorithm, Broyden method. Solving eigenvalue problems: power method, inverse iteration, translation, QR method. Interpolation and approximation problems: Lagrange and Hermite interpolation, spline interpolation, Chebyshev-approximation. Quadrature rules: Newton-Cotes formulas, Gauss quadrature. Numerical methods for ordinary differential equations: Euler method, Runge-Kutta methods, finite-difference methods, finite element method.

**Literature**

*Compulsory:*

-

*Recommended:*

- Atkinson, K.E.: Elementary Numerical Analysis. John Wiley, New York, 1993.
- Lange, K.: Numerical analysis for statisticians. Springer, New York, 1999.
- Press, W.H. – Flannery, B.P. – Tenkolsky, S.A. – Vetterling, W.T.: Numerical recipes in C. Cambridge University Press, Cambridge, 1988.
- Engeln-Mullgens, G. – Uhling, F.: Numerical algorithms with C. Springer, Berlin, 1996.

**Schedule:**

*1st week* Features of computions by computer, error propagation. Some important matrix transformations for solving linear systems and eigenvalue problems.

*2nd week* Solution of system of linear equations: Gaussian elimination and its variants

*3rd week* Algorithms of the Gauss elimination and its operational comlexity. Pivoting.

*4th week* Decompositions of matrices: Schur complement, LU decomposition, LDU decomposition, Cholesky factorisation, QR factorisation of matrices.

*5th week* Iterative methods for solving linear systems: Gauss-Seidel iteration and its convergence

*6th week* Preconditioning. The gradient method and the conjugate gradient method

*7th week* Approximate solution of nonlinear equations: Newton method, local and global convergence, quasi-Newton method, Levenberg–Marquardt algorithm, Broyden-method

*8th week* Numerical methods for solving eigenvalue problems: power method and inverse iteration

*9th week* Numerical methods for solving eigenvalue problems: shift method, the QR algorithm

*10th week* Interpolation and approximation problems: Lagrange-interpolation, Hermite-interpolation. Spline interpolation. Error of the approximation. Tschebisev-approximation

*11th week* Numerical integration: Newton-Cotes formulas. Composite quadrature formulas

*12th week* Gauss quadrature. Existence, convergence, error estimation

*13th week* Numerical methods for solving initial value problems of ordinary differential equations: Euler method, Runge-Kutta method

*14th week* Numerical methods for solving boundary value problems of ordinary differential equations: finite difference methods, finite element method

**Requirements:**

*- for a grade*

The course ends in an **examination**. The minimum requirement for the examination is 50%. Based on the score of the exam the grade for the examination is given according to the following table

| Score | Grade |
|---|---|
| 0-49 | fail (1) |
| 50-59 | pass (2) |
| 60-74 | satisfactory (3) |
| 75-89 | good (4) |
| 90-100 | excellent (5) |

**Person responsible for course:** Dr. Borbála Fazekas, associate professor, PhD

**Lecturer:** Dr. Borbála Fazekas, associate professor, PhD

| **Title of course**: Numerical analysis<br>**Code**: TTMBG0209 | **ECTS Credit points:** 2 |
|---|---|
| **Type of teaching, contact hours**<br>- lecture: -<br>- practice: 2 hours/week<br>- laboratory: - | |
| **Evaluation:** practical | |
| **Workload (estimated), divided into contact hours:**<br>- lecture: -<br>- practice: 28 hours<br>- laboratory: -<br>- home assignment: -<br>- preparation for the tests: 32 hours<br>Total: 60 hours | |
| **Year, semester**: 2nd year, 2nd semester | |
| **Its prerequisite(s)**: TTMBE0102, TTMBE0203 | |
| **Further courses built on it**: - | |

**Topics of course**

Features of computions by computer, error propagation. Some important matrix transformations for solving linear systems and eigenvalue problems. Gaussian elimination and its variants: its algorithms, operational complexity, pivoting. Decompositions of matrices: Schur complement, LU decomposition, LDU decomposition, Cholesky decomposition, QR decomposition. Iterative methods for solving linear and nonlinear systems: Gauss-Seidel iteration, gradient method, conjugate gradient method, Newton method, local and global convergence, quasi-Newton method, Levenberg–Marquardt algorithm, Broyden method. Solving eigenvalue problems: power method, inverse iteration, translation, QR method. Interpolation and approximation problems: Lagrange and Hermite interpolation, spline interpolation, Chebyshev-approximation. Quadrature rules: Newton-Cotes formulas, Gauss quadrature. Numerical methods for solving ordinary differential equations: Euler method, Runge-Kutta methods, finite-difference methods, finite element method.

**Literature**

*Compulsory:*

-

*Recommended:*

- Atkinson, K.E.: Elementary Numerical Analysis. John Wiley, New York, 1993.
- Lange, K.: Numerical analysis for statisticians. Springer, New York, 1999.
- Press, W.H. – Flannery, B.P. – Tenkolsky, S.A. – Vetterling, W.T.: Numerical recipes in C. Cambridge University Press, Cambridge, 1988.
- Engeln-Mullgens, G. – Uhling, F.: Numerical algorithms with C. Springer, Berlin, 1996.

**Schedule:**

*1st week* Features of computions by computer, error propagation. Some important matrix transformations for solving linear systems and eigenvalue problems. Solution of system of linear equations: Gaussian elimination and its variants

*2nd week* Algorithms of the Gauss elimination and its operational complexity. Pivoting.

*3rd week* Decompositions of matrices: Schur complement, LU decomposition, LDU decomposition, Cholesky factorisation, QR factorisation of matrices

*4th week* Iterative methods for solving linear systems: Gauss-Seidel iteration and its convergence

*5th week* Preconditioning. The gradient method and the conjugate gradient method

*6th week* Approximate solution of nonlinear equations: Newton method, local and global convergence, quasi-Newton method, Levenberg–Marquardt algorithm, Broyden-method

*7th week* Test

*8th week* Numerical methods for solving eigenvalue problems: power method and inverse iteration. Shift method, the QR algorithm

*9th week* Interpolation and approximation problems: Lagrange-interpolation, Hermite-interpolation. Spline interpolation. Error of the approximation. Tschebisev-approximation

*10th week* Numerical integration: Newton-Cotes formulas. Composite quadrature formulas

*11th week* Gauss quadrature. Existence, convergence, error estimation

*12th week* Numerical methods for solving initial value problems of ordinary differential equations: Euler method, Runge-Kutta method

*13th week* Numerical methods for solving boundary value problems of ordinary differential equations: finite difference methods, finite element method

*14th week* Test

**Requirements:**

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. A student can't make up any practice with another group. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence. In case of further absences, a medical certificate needs to be presented. Active participation is evaluated by the teacher in every class. If a student's behavior or conduct doesn't meet the requirements of active participation, the teacher may evaluate his/her participation as an absence because of the lack of active participation in class.

During the semester there are two tests: one test in the 7th week and the other test in the 14th week. The minimum requirement for the tests respectively is 50%. Based on the score of the tests, the grade for the tests is given according to the following table:

| Score | Grade |
|---|---|
| 0-49 | fail (1) |
| 50-59 | pass (2) |
| 60-74 | satisfactory (3) |
| 75-89 | good (4) |
| 90-100 | excellent (5) |

If the score of any test is below 50, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

**Person responsible for course:** Dr. Borbála Fazekas, associate professor, PhD

**Lecturer:** Dr. Borbála Fazekas, associate professor, PhD

| **Title of course**: Economic mathematics<br>**Code**: TTMBE0211 | **ECTS Credit points:** 3 |
|---|---|
| **Type of teaching, contact hours**<br>- lecture: 2 hours/week<br>- practice: -<br>- laboratory: - | |
| **Evaluation:** exam | |
| **Workload (estimated), divided into contact hours:**<br>- lecture: 28 hours<br>- practice: -<br>- laboratory: -<br>- home assignment: -<br>- preparation for the exam: 62 hours<br>Total: 90 hours | |
| **Year, semester**: 3rd year, 1st semester | |
| **Its prerequisite(s)**: TTMBE0211 | |
| **Further courses built on it**:- | |

**Topics of course**

Computation of future and present values, discounted present value and investment projects. Bounds for the budget, change of the budget line, consumer preferences, preference order. Indifference curves, marginal rate of substitution, utility, utility functions, Cobb-Douglas preferences, marginal utility. Optimal choice, consumer demand, demand curves, inverse demand curve, market demand, elasticity. Demands of constant elasticity, elasticity and marginal revenue, marginal revenue curves, income elasticity. Production functions, marginal rate of substitution. CES property, Cobb–Douglas type production function and its properties, Arrow–Chenery–Minhas–Solow type production function. Equilibrium points of two-person games, the best response mapping, game theoretic model of Bertrand and Cournot type duopoly. Individual and social preferences, social welfare function. Arrow's impossibility theorem. Consistent aggregation, bisymmetry equation. Influencing the distribution of incomes, the discounted present value of continuous income stream, Lorenz curve, Gini coefficient. Leontieff models.

**Literature**

*Compulsory:*

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*Recommended:*


- M. Carter: Foundations of Mathematical Economics, MIT Press, 2001.
- K. Sydsaeter, P. Hammond, Mathematics for Economic Analysis, Pearson Publishing, 1995.
- H. R. Varian: Intermediate Microeconomics: A Modern Approach, W.W. Norton, 1987.


**Schedule:**

*1st week*

Computation of future and present values, discounted present value and investment projects.

*2nd week*

Bounds for the budget, change of the budget line, consumer preferences, preference order.

*3rd week*

Indifference curves, marginal rate of substitution, utility, utility functions, Cobb-Douglas preferences, marginal utility.

*4th week*

Optimal choice, consumer demand, demand curves, inverse demand curve, market demand, elasticity.

*5th week*

Demands of constant elasticity, elasticity and marginal revenue, marginal revenue curves, income elasticity.

*6th week*

Production functions, marginal rate of substitution.

*7th week*

CES property, Cobb–Douglas type production function and its properties, Arrow–Chenery–Minhas–Solow type production function.

*8th week*

Equilibrium points of two-person games, the best response mapping, game theoretic model of Bertrand and Cournot type duopoly.

*9th week*

Individual and social preferences, social welfare function.

*10th week*

Arrow's impossibility theorem.

*11th week*

Consistent aggregation, bisymmetry equation.

*12th week*

Influencing the distribution of incomes, the discounted present value of continuous income stream,

*13th week*

Lorenz curve, Gini coefficient.

*14th week*

Leontieff models.

**Requirements:**
Attendance at **lectures** is recommended, but not compulsory.

*- for a grade*

The course ends in an **examination**. The minimum requirement for the examination is 50%. Based on the score of the exam the grade for the examination is given according to the following table:

| Score | Grade |
|---|---|
| 0-49 | fail (1) |
| 50-62 | pass (2) |
| 63-76 | satisfactory (3) |
| 77-88 | good (4) |
| 89-100 | excellent (5) |

**Person responsible for course:** Dr. Fruzsina Mészáros, assistant professor, PhD

**Lecturer:** Dr. Fruzsina Mészáros, assistant professor, PhD

| **Title of course**: Economic mathematics<br>**Code**: TTMBG0211 | **ECTS Credit points: 2** |
|---|---|
| **Type of teaching, contact hours**<br>- lecture: -<br>- practice: 2 hours/week<br>- laboratory: - | |
| **Evaluation:** practical | |
| **Workload (estimated), divided into contact hours:**<br>- lecture: -<br>- practice: 28 hours<br>- laboratory: -<br>- home assignment: -<br>- preparation for the tests: 32 hours<br>Total: 60 hours | |
| **Year, semester**: 3rd year, 1st semester | |
| **Its prerequisite(s)**: TTMBE0211 | |
| **Further courses built on it**:- | |

**Topics of course**

Computation of future and present values, discounted present value and investment projects. Bounds for the budget, change of the budget line, consumer preferences, preference order. Indifference curves, marginal rate of substitution, utility, utility functions, Cobb-Douglas preferences, marginal utility. Optimal choice, consumer demand, demand curves, inverse demand curve, market demand, elasticity. Demands of constant elasticity, elasticity and marginal revenue, marginal revenue curves, income elasticity. Production functions, marginal rate of substitution. CES property, Cobb–Douglas type production function and its properties, Arrow–Chenery–Minhas–Solow type production function. Equilibrium points of two-person games, the best response mapping, game theoretic model of Bertrand and Cournot type duopoly. Individual and social preferences, social welfare function. Arrow's impossibility theorem. Consistent aggregation, bisymmetry equation. Influencing the distribution of incomes, the discounted present value of continuous income stream, Lorenz curve, Gini coefficient. Leontieff models.

**Literature**

*Compulsory:*

-

*Recommended:*

- M. Carter: Foundations of Mathematical Economics, MIT Press, 2001.
- K. Sydsaeter, P. Hammond, Mathematics for Economic Analysis, Pearson Publishing, 1995.
- H. R. Varian: Intermediate Microeconomics: A Modern Approach, W.W. Norton, 1987.

**Schedule:**

*1st week*

Computation of future and present values, discounted present value and investment projects.

*2nd week*

Bounds for the budget, change of the budget line, consumer preferences, preference order.

*3rd week*

Indifference curves, marginal rate of substitution, utility, utility functions, Cobb-Douglas preferences, marginal utility.

*4th week*

Optimal choice, consumer demand, demand curves, inverse demand curve, market demand, elasticity.

*5th week*

Demands of constant elasticity, elasticity and marginal revenue, marginal revenue curves, income elasticity.

*6th week*

Production functions, marginal rate of substitution.

*7th week*

CES property, Cobb–Douglas type production function and its properties, Arrow–Chenery–Minhas–Solow type production function.

*8th week*

Equilibrium points of two-person games, the best response mapping, game theoretic model of Bertrand and Cournot type duopoly.

*9th week*

Individual and social preferences, social welfare function.

*10th week*

Arrow’s impossibility theorem.

*11th week*

Consistent aggregation, bisymmetry equation.

*12th week*

Influencing the distribution of incomes, the discounted present value of continuous income stream,

*13th week*

Lorenz curve, Gini coefficient.

*14th week*

Leontieff models.

**Requirements:**

*- for a practical*

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. A student can’t make up any practice with another group. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence. In case of further absences, a medical certificate needs to be presented. Active participation is evaluated by the teacher in every class. If a student’s behaviour or conduct doesn’t meet the requirements of active participation, the teacher may evaluate his/her participation as an absence because of the lack of active participation in class.

During the semester there are two tests: one test in the 7th week and the other test in the 14th week. The minimum requirement for the tests respectively is 50%. Based on the score of the tests, the grade for the tests is given according to the following table:

| Score | Grade |
|---|---|
| 0-49 | fail (1) |
| 50-62 | pass (2) |
| 63-76 | satisfactory (3) |
| 77-88 | good (4) |

| | |
|---|---|
| 89-100 | excellent (5) |

If the score of any test is below 50, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

**Person responsible for course:** Dr. Fruzsina Mészáros, assistant professor, PhD

**Lecturer:** Dr. Fruzsina Mészáros, assistant professor, PhD

| **Title of course**: Analysis with computer<br>**Code**: TTMBG0210 | **ECTS Credit points:** 4 |
|---|---|
| **Type of teaching, contact hours**<br>- lecture: -<br>- practice: -<br>- laboratory: 3 hours/week | |
| **Evaluation:** mid-semester grade | |
| **Workload (estimated), divided into contact hours:**<br>- lecture: -<br>- practice: -<br>- laboratory: 42 hours<br>- home assignment: -<br>- preparation for the test: 78 hours<br>Total: 120 hours | |
| **Year, semester**: 3rd year, 2nd semester | |
| **Its prerequisite(s)**: TTMBE0203 | |
| **Further courses built on it**: - | |

**Topics of course**

The Maple; types of data, simple for-cycles, defining functions. Examination of functions; continuity, limit, zeros, extrema, constrained extrema. Differentiation, integration and numerical integration. Programming of simple quadrature rules. Solving differential equations with analytic methods and visualizing the solutions. Solving differential equations with numerical methods, programming Runge–Kutta fomulas. Ways of defining vectors and matrces. Vector and matrix operations, decompositions of matrices. Solving linear systems of equations with direct and iterative methods. Graphic tools in two dimension: making figures, rotation, reflection, parametrized curves. Graphic tools in three dimension: functions of two variables, space curves, surfaces, solid figures. Making animations, illustrating geometric and physical problems. Curve fitting; Lagrange and Hermite interpolation, Bezier curves and spline interpolation. For-cycle and while-cycle, conditional branches. Writing simple procedures: searching for primes, recursive functions, divisibility problems. Writing complex procedures: numerical differentiation and integration, approximation of functions, orthogonal polynomials and differential equations.

**Literature**

*Compulsory: -*

*Recommended:*

- W. Gander, J. Hrebicek: Solving Problems in Scientific Computing Using Maple and MATLAB. Springer-Verlag, Berlin, Heidelberg, New York, 1993, 1995.

**Schedule:**

*1st week* Introduction. Data types of Maple: simple data types, complex data types.

*2nd week* Simple for-cycles. Defining functions. Examination of functions; continuity, limit, zeros, extrema, constrained extrema.

*3rd week* Differentiation, integration and numerical integration. Programming of simple quadrature rules.

*4th week* Solving differential equations with analytic methods and visualizing the solutions. Solving differential equations with numerical methods, programming simple Runge–Kutta fomulas.

*5th week* Ways of defining vectors and matrces. Vector and matrix operations, decompositions of matrices.

*6th week* Solving linear systems of equations with direct and iterative methods. Programming of simple iterative methods.

*7th week* Graphic tools in two dimension: making figures, rotation, reflection, parametrized curves.

*8th week* Graphic tools in three dimension: functions of two variables, space curves, surfaces, solid figures.

*9th week* Making animations, illustrating geometric and physical problems.

*10th week* Curve fitting; Lagrange and Hermite interpolation, Bezier curves and spline interpolation.

*11th week* For-cycle and while-cycle, conditional branches.

*12th week* Writing simple procedures: sequences, Taylor-series, extrema of functions.

*13th week* Writing complex procedures: numerical differentiation and integration, approximation of functions, orthogonal polynomials and differential equations.

*14th week* Test

**Requirements:**

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. A student can't make up any practice with another group. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence. In case of further absences, a medical certificate needs to be presented. Active participation is evaluated by the teacher in every class. If a student's behavior or conduct doesn't meet the requirements of active participation, the teacher may evaluate his/her participation as an absence because of the lack of active participation in class.

During the semester there is one test in the 14th week.

The minimum requirement for the test is 50%. Based on the score of the test, the grade for the test is given according to the following table:

| Score | Grade |
|---|---|
| 0-49 | fail (1) |
| 50-59 | pass (2) |
| 60-74 | satisfactory (3) |
| 75-89 | good (4) |
| 90-100 | excellent (5) |

If the score of the test is below 50, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

**Person responsible for course:** Dr. Borbála Fazekas, assistant professor, PhD

**Lecturer:** Dr. Borbála Fazekas, assistant professor, PhD

| **Title of course**: Computer geometry<br>**Code**: TTMBG0308 | **ECTS Credit points: 3** |
|---|---|

**Type of teaching, contact hours**
- lecture: -
- practice: 3 hours/week
- laboratory: -

**Evaluation:** mid-semester grade

**Workload (estimated), divided into contact hours:**
- lecture: -
- practice: 42 hours
- laboratory: -
- home assignment: -
- preparation for the exam: 48 hours

Total: 90 hours

**Year, semester**: 2nd year, 1st semester

**Its prerequisite(s)**: TTMBE0302

**Further courses built on it**: -

**Topics of course**

Analytical tools of descriptive geometry: analytical geometry of projections, oblique and orthogonal axonometry, central projection, central axonometry. Curves and surfaces. Hermite, Bézier curves and surfaces, B-splines. Representation of polyhedra.

**Literature**

*Recommended:*
- M. K. Agoston. Computer Graphics and Geometric Modeling. Springer-Verlag London Limited, 2005 ISBN 978-1-85233-818-3
- G. Farin. Curves and surfaces for computer-aided geometric design. Morgan Kaufmann, 5th edition, 2002 ISBN 978-1-55860-737-8

**Schedule:**

*1st week*

Basics of computer graphics I.

*2nd week*

Basics of computer graphics II.

*3rd week*

Realization of affine transformations.

*4th week*

Plotting functions of one variable.

*5th week*

Plotting curves in the plane.

*6th week*

Projections.

*7th week*

Representation of convex polyhedra.

*8th week*

Representation of surfaces.

*9th week*

Models of curves, Hermite curves.

*10th week*

Models of curves, Bézier curves.

*11th week*

Spline interpolation

*12th week*

Models of surfaces, Hermite and Bézier surfaces

*13th week*

B-spline surfaces

*14th week*

Representation of fractals

**Requirements:**

*- for a signature*

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence.

*- for a grade*

During the semester there are two tests. The minimum requirement for a grade is to get 50% of the total score of the two tests. The grade is given according to the following table:

| Score | Grade |
|---|---|
| 0-49 | fail (1) |
| 50-62 | pass (2) |
| 63-74 | satisfactory (3) |
| 75-86 | good (4) |
| 87-100 | excellent (5) |

If the score of any test is below 50, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

**Person responsible for course:** Dr. Ábris Nagy, assistant lecturer, PhD

**Lecturer:** Dr. Ábris Nagy, assistant lecturer, PhD

| **Title of course**: Linear programming<br>**Code**: TTMBE0607 | **ECTS Credit points:** 3 |
|---|---|
| **Type of teaching, contact hours**<br>- lecture: 2 hours/week<br>- practice: -<br>- laboratory: - | |
| **Evaluation:** exam | |
| **Workload (estimated), divided into contact hours:**<br>- lecture: 28 hours<br>- practice: -<br>- laboratory: -<br>- home assignment: -<br>- preparation for the exam: 62 hours<br>Total: 90 hours | |
| **Year, semester**: 2nd year, 1st semester | |
| **Its prerequisite(s)**: TTMBE0102 | |
| **Further courses built on it**: - | |

**Topics of course**

Problems reducible to linear programming tasks. Extreme points of convex polyhedra, simplex algorithm and its geometry, sensitivity analysis. Duality. Transportation and assignment model, network models. Special linear programming models.

**Literature**

*Compulsory:*

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*Recommended:*

- Vanderbei, R.: Linear Programming, Foundations and Extensions, Kluwer Academic Publishers, 1998.
- Bertsimas, D.; Tsitsiklis, J.: Introduction to Linear Optimization, Athena Scientific Series in Optimization and Neural Computation, 6, Athena Scientific, Belmont, 1997.

**Schedule:**

*1st week*

Introduction: The standard maximum and minimum problems, the diet problem, the optimal assignment problem

*2nd week*

Linear programming problems, the simplex method

*3rd week*

Degeneracy, lexicographic simplex method.

*4th week*

Effectiveness, number of steps, worst case, average case.

*5th week*

Duality I., special case, weak duality theorem

*6th week*

Duality II., strong duality theorem, dual simplex method

*7th week*

Matrix form, simplex tableau

*8th week*

primal and dual simplex methods.

*9th week*

Generalized problem to standard case.

*10th week*

Geometry of the simplex method

*11th week*

The transportation problem I.

*12th week*

The transportation problem II.

*13th week*

Assignment problem I.

*14th week*

Assignment problem II.

**Requirements:**

Attendance at **lectures** is recommended, but not compulsory.

*- for a grade*

The course ends in an **examination**. The minimum requirement for the examination is 50%. Based on the score of the exam the grade for the examination is given according to the following table:

| Score | Grade |
|---|---|
| 0-49 | fail (1) |
| 50-62 | pass (2) |
| 63-76 | satisfactory (3) |
| 77-88 | good (4) |
| 89-100 | excellent (5) |

**Person responsible for course:** Dr. Fruzsina Mészáros, senior assistant professor, PhD

**Lecturer:** Dr. Fruzsina Mészáros, senior assistant professor, PhD

| **Title of course**: Linear programming<br>**Code**: TTMBG0607 | **ECTS Credit points:** 2 |
|---|---|

**Type of teaching, contact hours**
- lecture: -
- practice: 2 hours/week
- laboratory: -

**Evaluation:** mid-semester grade

**Workload (estimated), divided into contact hours:**
- lecture: -
- practice: 28 hours
- laboratory: -
- home assignment: -
- preparation for the tests: 32 hours

Total: 60 hours

**Year, semester**: 2nd year, 1st semester

**Its prerequisite(s)**: TTMBE0102

**Further courses built on it**:-

**Topics of course**

Problems reducible to linear programming tasks. Extreme points of convex polyhedra, simplex algorithm and its geometry, sensitivity analysis. Duality. Transportation and assignment model, network models. Special linear programming models.

**Literature**

*Compulsory:*

-

*Recommended:*

- Vanderbei, R.: Linear Programming, Foundations and Extensions, Kluwer Academic Publishers, 1998.
- Bertsimas, D.; Tsitsiklis, J.: Introduction to Linear Optimization, Athena Scientific Series in Optimization and Neural Computation, 6, Athena Scientific, Belmont, 1997.

**Schedule:**

*1st week*

Introduction: The standard maximum and minimum problems, the diet problem, the optimal assignment problem

*2nd week*

Linear programming problems, the simplex method

*3rd week*

Degeneracy, lexicographic simplex method.

*4th week*

Effectiveness, number of steps, worst case, average case.

*5th week*

Duality I., special case, weak duality theorem

*6th week*

Duality II., strong duality theorem, dual simplex method

*7th week*

Matrix form, simplex tableau

*8th week*

primal and dual simplex methods.

*9th week*

Generalized problem to standard case.

*10th week*

Geometry of the simplex method

*11th week*

The transportation problem I.

*12th week*

The transportation problem II.

*13th week*

Assignment problem I.

*14th week*

Assignment problem II.

**Requirements:**

*- for a practical*

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. A student can't make up any practice with another group. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence. In case of further absences, a medical certificate needs to be presented. Active participation is evaluated by the teacher in every class. If a student's behavior or conduct doesn't meet the requirements of active participation, the teacher may evaluate his/her participation as an absence because of the lack of active participation in class.

During the semester there are two tests: one test in the 7th week and the other test in the 14th week. The minimum requirement for the tests respectively is 50%. Based on the score of the tests, the grade for the tests is given according to the following table:

| Score | Grade |
|---|---|
| 0-49 | fail (1) |
| 50-62 | pass (2) |
| 63-76 | satisfactory (3) |
| 77-88 | good (4) |
| 89-100 | excellent (5) |

If the score of any test is below 50, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

**Person responsible for course:** Dr. Fruzsina Mészáros, senior assistant professor, PhD

**Lecturer:** Dr. Fruzsina Mészáros, senior assistant professor, PhD

| **Title of course**: Nonlinear optimization<br>**Code**: TTMBE0608-EN | **ECTS Credit points: 3** |
|---|---|

**Type of teaching, contact hours**

- lecture: 2 hours/week
- practice: -
- laboratory: -

**Evaluation:** exam

**Workload (estimated), divided into contact hours:**

- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 62 hours

Total: 90 hours

**Year, semester**: 3rd year, 2nd semester

**Its prerequisite(s)**: **TTMBE0204**

**Further courses built on it**:

**Topics of course**

Normed and Banach spaces. Spaces of linear and multilinear functions. Basic elements of differential calculus in normed spaces. Gateaux and Fréchet derivatives and their calculus. Strong and continuous Gateaux and Fréchet differentiability and relations between them. Inverse function theorem. The Fermat principle and Lagrange's multiplier theorem concerning extremum problems. Higher order Gateaux and Fréchet differentiability. Young's theorem and Taylor's theorem. Second order necessary and sufficient conditions for extremum problems. The first order basic problems of the calculus of variations with weak and strong extremum. Computing the derivative of functionals. Du Bois–Reymond lemma. Euler–Lagrange's first order necessary condition and second order necessary and sufficient conditions for weak extremum. The higher order basic problems of the calculus of variations and the Euler–Lagrange equation concerning them. Weierstrass's necessary and sufficient conditions for strong extremum.

**Literature**

*Compulsory:-*

*Recommended:*

Dacorogna, B.: Introduction to the Calculus of Variations, Imperial College Press, London, 2014.

Durea, M.; Strugariu, R.: An Introduction to Nonlinear Optimization Theory, De Gruyter Open, Berlin, 2014.

Ioffe, A.D.; Tihomirov, V. M.: Theory of Extremal Problems, Studies in Mathematics and its Applications, 6., North-Holland Publishing Co., Amsterdam-New York, 1979.

Jahn, J.: Introduction to the Theory of Nonlinear Optimization, Springer Verlag, Berlin, 2007.

**Schedule:**

*1st week* Normed and Banach spaces. Norm on product of normed spaces. Boundedness and continuity of linear and multilinear maps. Properties of the norm. The structure of the space of linear and multilinear maps. Characterization of completeness. Dual space of normed spaces.

*2nd week* The directional, Gateaux, Hadamard and Fréchet derivative of maps acting between normed spaces and the connections among them. Continuity and differentiability.

*3rd week* Basic rules of differentiation: the sum rule, the derivative of multilinear maps and of Cartesian product. The chain rule and its consequences.

*4th week* Mean value inequality. Strong and continuous Gateaux, Hadamard and Fréchet differentiability and their relations. Local Lipschitz property and Hadamard differentiability. Partial derivative with respect to a subspace. The connection of partial differentiability and Gateaux, Hadamard and Fréchet differentiability.

*5th week* The inverse and implicit function theorems for strongly Fréchet differentiable maps. Local minimum and maximum, the Fermat principle. Constrained optimization and the Lagrange multiplier rule.

*6th week* Higher-order Gateaux, Hadamard and Fréchet derivatives, the Young theorem about the symmetry of higher-order derivatives and the Taylor theorem.

*7th week* Characterizations of positive semidefinite, positive definite and strongly positive definite bilinear forms. The second-order necessary and sufficient conditions of optimality for problems with C^2 data.

*8th week* The space of k times continuously differentiable maps and its equivalent norms. Gateaux and Fréchet derivative of nonlinear functions given in terms of integrals.

*9th week* The fundamental problem of the calculus of variations for single variable functions. Admissible functions and the notion of weak and strong optimum.

*10th week* The Du Bois–Reymond lemma. The Euler–Lagrange equation for the first-order weak optimum problem of the calculus of variations.

*11th week* Second-order necessary and sufficient conditions for the first-order weak optimum problem of the calculus of variations: The Legendre and Jacobi conditions.

*12th week* The generalized Du Bois–Reymond lemma. The Euler–Lagrange equation for the higher-order weak optimum problem of the calculus of variations.

*13th week* The strong optimum problem of the calculus of variations. The Weierstrass E function. The necessary and sufficient condition of Weierstrass for the strong optimum problem of the calculus of variations.

*14th week* The brachistochrone problem, the chain-curve problem, and the problem of minimal rotation invariant surface.

**Requirements:**

*- for a signature*

Attendance at **lectures** is recommended, but not compulsory.

*- for a grade*

The course ends in an **examination**. Before the examination students must have grade at least ‘pass’ on *Nonlinear optimization* practice (TTMBG0608-EN). The grade for the examination is given according to the following table:

| Score | Grade |
|---|---|
| 0-49 | fail (1) |
| 50-61 | pass (2) |
| 62-74 | satisfactory (3) |
| 75-87 | good (4) |
| 88-100 | excellent (5) |

If the average of the score of the examination is below 50, students can take a retake examination in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

**Person responsible for course:** Prof. Dr. Zsolt Páles, university professor, DSc

**Lecturer:** Prof. Dr. Zsolt Páles, university professor, DSc

| **Title of course**: Nonlinear optimization<br>**Code**: TTMBG0608-EN | **ECTS Credit points: 2** |
|---|---|
| **Type of teaching, contact hours**<br>- lecture: -<br>- practice: 2 hours/week<br>- laboratory: - | |
| **Evaluation:** mid-term and end-term tests | |
| **Workload (estimated), divided into contact hours:**<br>- lecture: -<br>- practice: 28 hours<br>- laboratory: -<br>- home assignment: 16 hours<br>- preparation for the tests: 16 hours<br>Total: 60 hours | |
| **Year, semester**: 3rd year, 2nd semester | |
| **Its prerequisite(s)**: TTMBE0204 | |
| **Further courses built on it**: | |

**Topics of course**

Normed and Banach spaces. Spaces of linear and multilinear functions. Basic elements of differential calculus in normed spaces. Gateaux and Fréchet derivatives and their calculus. Strong and continuous Gateaux and Fréchet differentiability and relations between them. Inverse function theorem. The Fermat principle and Lagrange's multiplier theorem concerning extremum problems. Higher order Gateaux and Fréchet differentiability. Young's theorem and Taylor's theorem. Second order necessary and sufficient conditions for extremum problems. The first order basic problems of the calculus of variations with weak and strong extremum. Computing the derivative of functionals. Du Bois-Reymond lemma. Euler–Lagrange's first order necessary condition and second order necessary and sufficient conditions for weak extremum. The higher order basic problems of the calculus of variations and the Euler–Lagrange equation concerning them. Weierstrass's necessary and sufficient conditions for strong extremum.

**Literature**

*Compulsory:-*
*Recommended:*

Dacorogna, B.: Introduction to the Calculus of Variations, Imperial College Press, London, 2014.
Durea, M.; Strugariu, R.: An Introduction to Nonlinear Optimization Theory, De Gruyter Open, 2014.
Ioffe, A.D.; Tihomirov, V. M.: Theory of Extremal Problems, Studies in Mathematics and its Applications, 6., North-Holland Publishing Co., Amsterdam-New York, 1979.
Jahn, J.: Introduction to the Theory of Nonlinear Optimization, Springer Verlag, Berlin, 2007.

**Schedule:**

*1st week* Classical finite and infinite dimensional normed and Banach spaces. Sequence and functions spaces. The Hölder inequality. Comparison of norms, equivalent and non- equivalent norms. Computation of the norm of bounded linear and multilinear maps. Representation of linear functionals.

*2nd week* Computing the directional, Gateaux, Hadamard and Fréchet derivative of maps acting between finite and infinite dimensional normed spaces. Examples of functions with different differentiability properties.

*3rd week* Application of the basic rules of differentiation: the sum rule, the differentiation rule of multilinear maps and of the Cartesian product and the chain rule.

*4th week* Investigation of strong and continuous Gateaux, Hadamard and Fréchet differentiability.

Estimating the local Lipschitz modulus. Computation of partial derivative with respect to a subspace. Applications of the connection of partial differentiability, Gateaux, Hadamard and Fréchet differentiability. Examples of maps with different regularity properties.

*5th week* Applications of the inverse and implicit function theorems. Applications of the Fermat principle for local minimum and maximum and Lagrange multiplier rule for constrained optimization problems in finite and infinite dimensional settings.

*6th week* Computation of higher-order Gateaux, Hadamard and Fréchet derivatives and applications of the Taylor theorem. Investigations of second-order conditions of optimality.

*7th week* Mid-term test from problems of differential calculus in normed spaces.

*8th week* First and higher-order Gateaux and Fréchet derivative of nonlinear maps given in terms of integrals and boundary conditions.

*9th week* Investigation of problems of the calculus of variations for single variable functions with respect to weak and strong optimum. Examples for the non-existence of solutions.

*10th week* Applications of the Du Bois-Reymond lemma. Constructing and solving the Euler–Lagrange equation for the first-order weak optimum problem of the calculus of variations. Verification of optimality in the presence of convexity properties.

*11th week* Applications and verification of the Legendre and Jacobi conditions for the first-order weak optimum problem of the calculus of variations.

*12th week* Applications of the generalized Du Bois-Reymond lemma. Constructing and solving the corresponding Euler–Lagrange equation for the higher-order weak optimum problem of the calculus of variations.

*13th week* Constructing *t*he Weierstrass E function for the strong optimum problem of the calculus of variations. Verifying the necessary and sufficient conditions of Weierstrass.

*14th week* End-term test from problems of calculus of variations.

**Requirements:**

*- for a signature*

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. A student can't make up any practice with another group. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence. In case of further absences, a medical certificate needs to be presented. Missed practice classes should be made up for at a later date, to be discussed with the tutor. Active participation is evaluated by the teacher in every class. If a student's behaviour or conduct doesn't meet the requirements of active participation, the teacher may evaluate his/her participation as an absence because of the lack of active participation in class.

During the semester there are two tests: the mid-term test in the 7th week and the end-term test in the 14th week. Students have to sit for the tests.

*- for a grade*

The minimum requirement for the average of the mid-term and end-term tests is 50%.

| Score | Grade |
|---|---|
| 0-49 | fail (1) |
| 50-61 | pass (2) |
| 62-74 | satisfactory (3) |
| 75-87 | good (4) |
| 88-100 | excellent (5) |

If the average of the scores of the tests is below 50, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

**Person responsible for course:** Prof. Dr. Zsolt Páles, university professor, DSc

**Lecturer:** Prof. Dr. Zsolt Páles, university professor, DSc

| **Title of course**: Basics of mathematics<br>**Code**: TTMBG0001 | **ECTS Credit points:** 0 |
|---|---|

**Type of teaching, contact hours**
- lecture: -
- practice: 1 hours/week
- laboratory: -

**Evaluation:** signature

**Workload (estimated), divided into contact hours:**
- lecture: -
- practice: 14 hours
- laboratory: -
- home assignment: -
- preparation for the exam: 14 hours

Total: 28 hours

**Year, semester**: 1st year, 1st semester

**Its prerequisite(s)**: -

**Further courses built on it**: -

**Topics of course**

Algebraic transformations. Solution of different type equations, equation systems, inequalities and inequality systems. Basic notions of trigonometry and coordinate geometry.

**Literature**

*Compulsory:*
-
*Recommended:*
A. Bérczes and Á. Pintér: College Algebra. University of Debrecen, 2013.
R. D. Gustafson: College algebra and trigonometry. Pacific Grove, Brooks/Cole, 1986.

**Schedule:**

*1st week*
Algebraic transformations, identities, simplification of rational algebraic expressions.

*2nd week*
Simplification of irrational algebraic expressions, rationalization of denominator.

*3rd week*
Parametric linear equations, equation systems.

*4th week*
Quadratic equations, equation systems.

*5th week*
Parametric quadratic equations.

*6th week*
Sign of linear and quadratic expressions, inequalities, inequality systems (table of signs).

*7th week*
Equations containing absolute value.

*8th week*

Trigonometry: geometric interpretation of trigonometric functions and basic properties.

*9th week*

Identities of sum and difference of angle and trigonometric identities.

*10th week*

Trigonometric equations, inequalities. Method of phase shift.

*11th week*

Coordinate geometry: lines and circles in a plane, intersectional exercises. Distance of points and of point and line.

*12th week*

Lines and circles in the plane, exercises concerning tangent line.

*13th week*

Exponential function and its inverse, the logarithm.

*14th week*

Exponential and logarithmic equations, inequality.

**Requirements:**

*- for a signature*

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

The course is evaluated on the basis of two written tests during the semester. The signature is given if the student obtains at least 60 percent of the total points.

If a student fail to pass at first attempt, then a retake of the tests is possible.

*- for a grade*

There is no grading in this course.

*-an offered grade:*

There is no grading in this course.

**Person responsible for course:** Dr. Nóra Györkös-Varga, assistant professor, PhD

**Lecturer:** Dr. Nóra Györkös-Varga, assistant professor, PhD